Title:	Carbon Footprint: Getting a Sense of the Numbers
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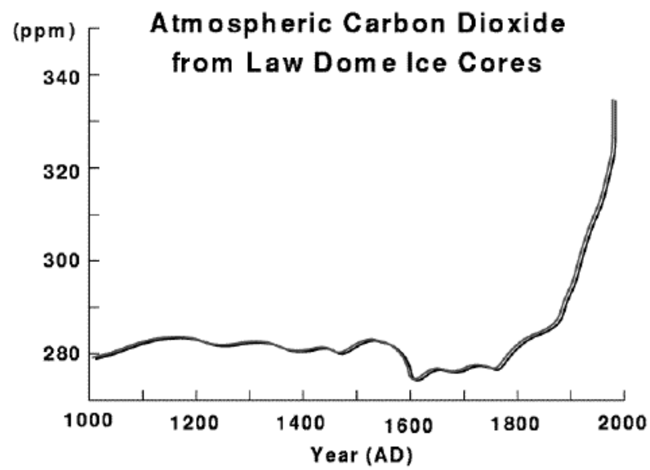
Module Summary:	In this module we explore carbon emissions and other modern waste in order to develop our number sense and work with units and dimension.
Target Audience:	High School and Liberal Arts Major College Students
Prerequisite Math:	High school Algebra I or equivalent
Math Fields:	Quantitative Literacy / Quantitative Reasoning
Technology:	Calculator, computer or mobile device with search capabilities
Applications:	Environmental Science
Goals:	As a result of this module students will: • Possess a deeper understanding of how our choices affect carbon emissions • Be able to identify information needed to solve dimension problems • Use data to answer questions and make predictions about carbon footprints • Be able to work with units and dimensional analyses • Be able to calculate percent error and percent change • Have a better understanding of the scale of modern household waste • Be able to use proportions and scale factors to solve problems • Be able to work with numbers in scientific notation • Be able to read a bar graph • Improve their skills finding information with a search engine • Improve their number sense and possess skills for understanding large numbers
Resources:	US Energy Information Administration website Google (for unit conversion factors) Textbooks: General chemistry text
Module:	The time required for students to complete this module will depend on the student's familiarity with the material. This is designed as a lab activity to review and solidify understanding of math processes that the students have already practiced, and to put these processes into a real-life context. The lab is intended to be used in the classroom individually or (even better) in small groups. It should be able to be completed in one class day, with the unfinished portion assigned as homework and no preparatory assignment required before students can begin the module. Further reading is provided for students who are interested in a more in-depth understanding of the underlying processes.

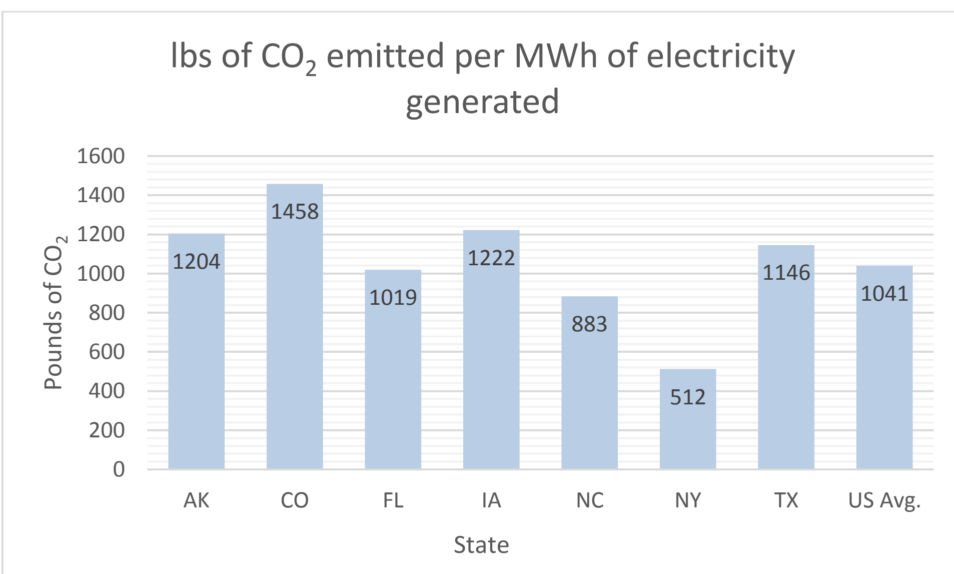
Carbon Footprint: Dimensions and Scale

Without Earth's atmospheric greenhouse effect, life as we know it could not exist on our planet. Greenhouse gasses act like a blanket to trap heat so that we can keep warm despite the cold of space. But as we have all discovered when trying to sleep comfortably in the summer, sometimes a blanket can be too warm. Carbon dioxide (CO_2) is a greenhouse gas that is produced when anything is burned for fuel. Since the beginning of the Industrial Revolution we have been extracting large amounts of fuel from deep underground and burning it, releasing the CO_2 into the air. You can see the change in atmospheric CO_2 levels over the past 1000 years in the chart below, which took its data from drilled ice cores.



CO_2 levels for the past millennium, graph from Elmhurst College Chemistry department.

Internal combustion engines, such as those in cars, burn fuel directly. When we use electricity in our homes and workplaces we don't see the fuels being burned, but most electricity still comes from burning fossil fuels at the power plant, rather than from sustainable sources. The amount of CO_2 released when electricity is produced depends on what type of power plants are in the area. You can see the average CO_2 emissions per megawatt-hour (MWh) of electricity generated in several states in the following graph (2016 data from the US EIA).



The appliances that we use in our homes measure their electricity usage in watts (W) and the total energy consumption depends on how long we leave them on. For example, a 60 W light bulb left on for two hours uses $60\ W * 2\ h = 120\ Wh$. The usual metric prefixes apply to this unit of energy consumption, Watt-hours (Wh). It will be helpful to note that 1 lb of CO_2 fills 8.57 ft^2 of volume.

Part I

1. The expression below shows the units of four quantities multiplied together. Determine the units of the resulting quantity.

$$\frac{meters}{seconds} * \frac{miles}{meters} * \frac{seconds}{minutes} * \frac{minutes}{hours} =$$

2. The equation below shows only the units of each quantity involved. Determine the units for the missing quantity.

    a. $\frac{acres}{tons} * \underline{\hspace{6cm}} * \frac{pounds}{person*day} * \frac{days}{year} = \frac{acres}{person*year}$

    b. How would you write or say the unit $\frac{acres}{person*year}$ in words?

3. We have choices when buying light bulbs. A 60 Watt incandescent bulb, a 13 Watt compact fluorescent bulb (CFL), and a 3 Watt light-emitting diode (LED) all produce the same amount of light. Refer to the graph on the pervious page to determine the CO_2 emitted by three people in Iowa, who each burn a different type of bulb 8 hours a day, every day, for a year. Give your answers in cubic feet.

    a. CO_2 emissions for the incandescent bulb:

    b. CO_2 emissions for CFL bulb:

    c. Can you predict the CO_2 emissions for the LED without redoing the whole calculation? What is the CO_2 emissions for the LED?

4. Now suppose that everyone in the United States operates an identical lamp such as in question 3 for 8 hours a day, every day, for a year. Use the graph to identify the average CO_2 emissions from electricity production in the United States and calculate the total carbon emissions for each type of bulb. Write your answers in scientific notation.

    a. Total CO_2 emissions if everyone used incandescent bulbs:

    b. Total CO_2 emissions if everyone used LEDs:

5. Use your answer from problem 4a to determine how many Superdomes this CO_2 would fill. The volume of the Superdome is 155 million cubic feet.

6. Which number is easier for you to understand and relate to, the answer from 4a in cubic feet, or the answer from 5 in terms of Superdomes? Write a few sentences explaining which you chose and why.

Part II

7. In 2016 the total carbon footprint of the United States was 5.7×10^9 tons of CO_2. Look up the US population for 2016. What was the average American's carbon footprint that year?

8. Forests absorb CO_2 from the air, and different types of forests absorb different amounts. But on average, an acre of forest can absorb 4.9 tons of carbon in a year. How many acres of forest are needed to absorb one average American's carbon emissions?

9. How many acres of forest are needed to absorb all of the CO_2 that the United States produces?

10. Look up the amount of forest in the United States. Before European colonization of North America this same region had 1.023×10^9 acres of forest. What is the percentage change in the amount of forest since colonization?

11. Do we have enough forest in the United States to absorb all of the CO_2 that we produce? If we still had all the forests that we had before Europeans arrived, would we have enough? What do you think happens with the excess – either the excess carbon or the excess forest? Write a paragraph to answer these questions and to explain what you think happens and why.

Part III

12. If you took all of the cardboard pizza boxes used in the United States over one year and stacked them one on top of the other, about how many miles do you think this stack would reach? Take a guess.

13. In order to calculate the height of the stack, what information will you need to know?

14. Look up the items in your list from question 13 and calculate the actual height of the stack.

15. Mount Everest is 5.5 miles high. How many times higher is the stack of pizza boxes that go to American landfills every year?

16. Write a couple of sentences to describe how your guess relates to the actual height of the stack.

Instructor Notes and Answer Key

Part I

1. The expression below shows the units of four quantities multiplied together. Determine the units of the resulting quantity.

$$\frac{meters}{seconds} * \frac{miles}{meters} * \frac{seconds}{minutes} * \frac{minutes}{hours} = \frac{\mathbf{miles}}{\mathbf{hour}}$$

2. The equation below shows only the units of each quantity involved. Determine the units for the missing quantity.

   We need the tons in the first fraction and the pounds in the third fraction to cancel out.

   a. $\frac{acres}{tons} * \frac{\mathbf{tons}}{\mathbf{pounds}} * \frac{pounds}{person*day} * \frac{days}{year} = \frac{acres}{person*year}$

   b. How would you write or say the unit $\frac{acres}{person*year}$ in words?

   Acres per person per year

3. We have choices when buying light bulbs. A 60 Watt incandescent bulb, a 13 Watt compact fluorescent bulb (CFL), and a 3 Watt light-emitting diode (LED) all produce the same amount of light. Refer to the graph on the pervious page to determine the CO_2 emitted by three people in Iowa, who each burn a different type of bulb 8 hours a day, every day, for a year. Give your answers in cubic feet.

   a. CO_2 emissions for the incandescent bulb:

   8 hours * 365 days = 2920 hours

$$\frac{60\ W*2920\ hrs}{year} * \frac{1\ MW}{1{,}000{,}000\ W} * \frac{1222\ lbs}{1\ MWh} * \frac{8.57\ ft^3}{1\ lb} = 1834.79\ ft^3 \text{ of } CO_2 \text{ per year}$$

   b. CO_2 emissions for CFL bulb:

$$\frac{13\ W*2920\ hrs}{year} * \frac{1\ MW}{1{,}000{,}000\ W} * \frac{1222\ lbs}{1\ MWh} * \frac{8.57\ ft^3}{1\ lb} = 397\ ft^3 \text{ of } CO_2 \text{ per year}$$

   c. Can you predict the CO_2 emissions for the LED without redoing the whole calculation? What is the CO_2 emissions for the LED?

   The only difference between the previous two calculations is that the 60W changed to 13W, so this can be solved easily by setting up a proportion: $\frac{60\ W}{1834.79\ ft^3} = \frac{3\ W}{\mathbf{91.74\ ft^3}}$

4. Now suppose that everyone in the United States operates an identical lamp such as in question 3 for 8 hours a day, every day, for a year. Use the graph to identify the average CO_2 emissions from electricity production in the United States and calculate the total carbon emissions for each type of bulb. Write your answers in scientific notation.

    a. Total CO_2 emissions if everyone used incandescent bulbs:

$$\frac{60\ W * 2920\ hrs}{year} * \frac{1\ MW}{1{,}000{,}000\ W} * \frac{1041\ lbs}{1\ MW} * \frac{8.57\ ft^3}{1\ lb} * \frac{325{,}700{,}000}{1} = \mathbf{5.09\ x\ 10^{11}\ ft^3}$$

    b. Total CO_2 emissions if everyone used LEDs:

$$\frac{60\ W}{5.09\ x\ 10^{11} ft^3} = \frac{3\ W}{\mathbf{2.55\ x\ 10^{10} ft^3}}$$

5. Use your answer from problem 4a to determine how many Superdomes this CO_2 would fill. The volume of the Superdome is 155 million cubic feet.

$$\frac{5.09\ x10^{11} ft^3}{155\ x\ 10^6 ft^3} = \mathbf{3{,}283.87}\ \textbf{superdomes}$$

6. Which number is easier for you to understand and relate to, the answer from 4a in cubic feet, or the answer from 5 in terms of Superdomes? Write a few sentences explaining which you chose and why.

    For most of us, it is easier to imagine how big a few thousand stadiums would be than to make sense of trillions of cubic feet. Even so, 3283 superdomes is still enormous. For further exploration you could compare the area they would cover to the size of various locations such as cities or islands. How does this compare to the size of Manhattan?

Part II

7. In 2016 the total carbon footprint of the United States was 5.7 x 10^9 tons of CO_2. Look up the US population for 2016. What was the average American's carbon footprint that year?

   The US population in 2016 was 232.4 x 10^6 people. $\frac{5.7\ x\ 10^9\ tons}{323.4\ x\ 10^6\ people} =$ **17.63 tons per person**

8. Forests absorb CO_2 from the air, and different types of forests absorb different amounts. But on average, an acre of forest can absorb 4.9 tons of carbon in a year. How many acres of forest are needed to absorb one average American's carbon emissions?

   $\frac{17.63\ tons}{person} * \frac{1\ acre}{4.9\ tons} =$ **3.60 acres per person**

9. How many acres of forest are needed to absorb all of the CO_2 that the United States produces?
   $\frac{3.6\ acres}{person} * 323.4\ x\ 10^6\ people =$ **1.16 x 10^9 acres**

10. Look up the amount of forest in the United States. Before European colonization of North America this same region had 1.023 x 10^9 acres of forest. What is the percentage change in the amount of forest since colonization?

    There are currently 747 million acres of forest, whereas in the past there were 1023 million acres.

    $$\frac{747 - 1023}{1023} = \mathbf{-26.98\%}$$

11. Do we have enough forest in the United States to absorb all of the CO_2 that we produce? If we still had all the forests that we had before Europeans arrived, would we have enough? What do you think happens with the excess – either the excess carbon or the excess forest? Write a paragraph to answer these questions and to explain what you think happens and why.

    We do not have enough forest to absorb all of our carbon emissions. If we still had all of the original forests we would come much closer to having enough, but still not quite be there. All of our extra CO_2 remains in the atmosphere and moves over other countries – can their forests absorb our excess? Unfortunately, no. While we have higher emissions per capita than other countries, we also have a much higher than average portion of our land forested. We don't have enough forests to absorb it all, and other countries don't, either.

Part III

12. Americans love pizza! According to *Packaged Facts* we each eat, on average, 46 slices per year, and much of that is delivery or takeout. If you took all of the cardboard pizza boxes used in the United States over one year and stacked them one on top of the other, about how many miles do you think this stack would reach? Take a guess.

    There are no wrong answers to this problem.

13. In order to calculate the height of the stack, what assumptions will you need to make, and what information will you need to know?

    Various answers could be correct, depending on your approach.
    The number of pizza boxes used in a year
    The number of slices in the average pizza
    What proportion of pizzas are transported in takeout boxes
    The height of an average pizza box in inches
    Conversions from inches to miles

14. Look up the items in your list from question 13 and calculate the actual height of the stack.

    According to webrestaurantstore.com pizza boxes are 1.75" high. Assuming that 75% of pizzas are transported in a box, and that the average pizza has eight slices, we have:

    $$\frac{1\ pizza}{8\ slices} * \frac{46\ slices}{person} * 325.7\ x\ 10^6\ people = 1.87 \text{ x } 10^9 \text{ total pizzas eaten}$$

    $.75 * 1.87 \text{ x } 10^9 = 1.4 \text{ x } 10^9$ pizza boxes used

    $$\frac{1.75\ inches}{box} * \frac{1.4\ x\ 10^9\ boxes}{1} * \frac{1\ foot}{12\ inches} * \frac{1\ mile}{5280\ feet} = \text{38,668 miles}$$

15. Mount Everest is 5.5 miles high. How many times higher is the stack of pizza boxes that go to American landfills every year?

    $$\frac{38{,}668\ miles}{5.5\ miles} = 7{,}010.5 \text{ times as high.}$$

16. Write a couple of sentences to describe how your guess relates to the actual height of the stack.

    Most of us will significantly underestimate this. 38,668 miles is almost 1/6 of the distance to the moon!