# Applied Combinatorics

## by Fred S. Roberts and Barry Tesman

## Answers to Selected Exercises[1]

### Chapter 5

***Section 5.1***.

**1(b)**. $\sum_{k=0}^{\infty} \frac{2^k x^k}{k!}$;

**1(e)**. $1 + x + \frac{3x^2}{2!} + \sum_{k=3}^{\infty} \frac{x^k}{k!}$;

**2(b)**. $\sum_{k=0}^{\infty} x^{k+2}$;

**2(c)**. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{8k+4} = x^4 - \frac{x^{12}}{3!} + \frac{x^{20}}{5!} - \cdots$;

**3(a)**. $1 + x + x^2$;

**3(d)**. $-x - x^2 + \frac{1}{1-x}$;

**3(j)**. $-1 - x + e^x$;

**4(c)**. $(0, 0, 0, 1, 1, 1, \ldots)$;

**4(h)**. $\left(6, \frac{2^1}{1!}, \frac{2^2}{2!}, \frac{2^3}{3!}, \ldots, \frac{2^k}{k!}, \ldots\right)$;

**4(k)**. $(1, 0, 1, 0, 1, 0, \ldots)$;

**5(a)**. 1;

**6(a)**. 0;

**11**. $3^n$;

**13(b)**. $1 + 8x + 12x^2$;

**14(b)**. $1 + 16x + 72x^2 + 96x^3 + 24x^4$.

***Section 5.2***.

**1(a)**. $\left(0, 0, 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots\right)$;

**1(b)**. $\left(1, 0, -\frac{1}{3!}, 0, \frac{1}{5!}, 0, \ldots\right)$;

[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

**1(e)**. $(9, 8, 5, 6, 5, 5, 5, \ldots)$;

**2(a)**. $(0, 1, 2, 3, \ldots)$;

**3(b)**. $a_k = \sum_{i=0}^{k} i$;

**4(a)**. $a_k = 15(k+1)$;

**5**. 5;

**6**. 38;

**7(a)**. $(1, 4, 9, 27, 81, \ldots)$;

**8(a)**. $A(x) + (11 - a_3)x^3$;

**10(a)**. $\frac{x}{(1-x)^2} + \frac{3}{1-x} = \frac{3-2x}{(1-x)^2}$;

**11(b)**. $\frac{2x}{(1-x)^3}$;

**17**. $R(x, B) = 1 + 6x + 11x^2 + 8x^3 + 2x^4$.

***Section 5.3***.

**1(a)**. $(1 + x + x^2)^2(1 + x + x^2 + x^3)^2$, coefficient of $x^5$;

**1(e)**. $(1 + x + x^2 + x^3 + x^4 + x^5 + x^6)^2(x^4 + x^5 + x^6 + x^7)$, coefficient of $x^{12}$;

**1(i)**. $(1 + x + x^2 + x^3 + x^4 + \cdots + x^{12})^8$, coefficient of $x^{12}$;

**1(m)**. $(1 + x + x^2 + x^3 + \cdots)(1 + x^5 + x^{10} + x^{15} + \cdots)(1 + x^{10} + x^{20} + x^{30} + \cdots)(1 + x^{25} + x^{50} + x^{75} + \cdots)$, coefficient of $x^{100}$;

**2**. $(1 + x)^3(1 + x + x^2 + \cdots)$, coefficient of $x^5$;

**8**. $(1 + x + x^2 + x^3 + x^4)^3(1 + x + x^2 + x^3)(1 + x + \cdots + x^7)(1 + x + \cdots + x^{12})$;

**13(c)**. $(1 + x)(1 + x^2)(1 + x^3) \cdots (1 + x^k)$.

***Section 5.4***.

**1(b)**. $-10$;

**2(a)**. 35;

**3**. $-\frac{1}{9}$;

**6**. $\binom{16}{11}$;

**11**. $\binom{12}{7}$;

**13(b)**. $\binom{p+k-1}{k}$;

**18(b)**. $\binom{n-1}{r-1}$;

**21**. 37.

***Section 5.5***.

**1(b)**. $e^{3x}$;

**1(c)**. $e^x - x - \frac{x^2}{2!}$;

**2(a)**. $a_k = 4k!$;

**2(g)**. $a_k = 2^k + 5^k$;

**6(a)**. $\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)^2 (1+x)^2$, coefficient of $\frac{x^3}{3!}$;

**6(d)**. $(1 + x + x^2 + \cdots + x^{2n})^3$, coefficient of $x^{3n}$;

**12**. $(e^x - 1)^p$;

**14**. $\frac{1}{2}[5^k - 3^k]$;

**19(b)**. $\left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right] 3!$.

***Section 5.6***.

**1(a)**. $G(x) = \frac{1}{3} + \frac{1}{3}x + \frac{1}{3}x^2, E = 1, V = \frac{2}{3}$;

**7(a)**. $\frac{px}{1+px-x}$;

**8(b)**. $E = \frac{qm}{p}, V = \frac{q^2 m}{p^2} + \frac{qm}{p}$.

***Section 5.7***.

**1(d)**. Coleman: $0, \frac{4}{8}, \frac{4}{8}, \frac{4}{8}$; Banzhaf: $0, \frac{4}{12}, \frac{4}{12}, \frac{4}{12}$;

**2(b)**. [5; 4, 2, 1, 1];

**4(c)**. $\frac{5}{12}, \frac{3}{12}, \frac{3}{12}, \frac{1}{12}$.