Applied Combinatorics

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Answers to Selected Exercises[1]

Chapter 8

Section 8.1.

1(c). reflexivity, transitivity;

1(h). symmetry;

2. Yes;

3(a). $\{a, b\}, \{c, d\}$;

7. $\{bb\}, \{rr\}, \{pp\}, \{bp, pb\}, \{br, rb\}, \{pr, rp\}$;

9. see Figure 8.10 in the text;

13(a). there are 3 others;

16. 3.

Section 8.2.

1(b). $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 4 & 1 & 3 \end{pmatrix}$;

2(a). $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$;

5(a). all;

5(f). **G1, G2**;

8(a). no;

9. $C(1) = \{1, 7\}$;

10(a). $\{1, 5\}, \{2, 4\}, \{3\}$;

11(a). reflection in a diagonal from upper right to lower left;

12(a). $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$;

[1]More solutions to come. Comments/Corrections would be appreciated and should be sent to: Barry Tesman (tesman@dickinson.edu) or Fred Roberts (froberts@dimacs.rutgers.edu).

21. no, for example, associativity may not hold. Let $p = 6$, then $(2 \circ 3) \circ 5 \neq 2 \circ (3 \circ 5)$;

23(a). $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \right\}$.

Section 8.3.

5(a). $\frac{1}{2}(5+1) = 3 : \{1,5\}, \{2,4\}, \{3\}$;

6(a). (a) $\mathrm{St}(1) = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \right\}$, **(b)** $C(1) = \{1,5\}$;

12. no, G is not a group of permutations.

13(b). 12.

Section 8.4.

2. 3^8;

4(a). black;

5(a). r;

6(a). no;

6(g). 8;

7(a). no;

7(f). 4;

10(e). 6;

21(c). if π_i is $\begin{pmatrix} \{1,2\} & \{1,3\} & \{2,3\} \\ \{1,2\} & \{2,3\} & \{1,3\} \end{pmatrix}$, then $\mathrm{Inv}(\pi_i^*)$ is 4.

Section 8.5.

1(a). (2)(5)(17)(364);

1(b). (2)(1534);

1(c). (164827)(35);

3(a). $\mathrm{cyc}(\pi)$ is 5 and 3, respectively;

4(a). x_1^5 and $x_1 x_2^2$, respectively;

5(a). $\frac{1}{2}(x_1^5 + x_1 x_2^2)$;

9(a). $\frac{1}{2}\left[2^4 + 2^2\right] = 10$;

12(a). $\frac{1}{2}\left[2^3 + 2^2\right] = 6$;

22(b). x_2^4;

26. (16)(15)(14)(13)(12);

31(a). $D_5 = (2-1)(3-2)(3-1)(4-3)(4-2)(4-1)(5-4)(5-3)(5-2)(5-1)$;

31(a). $\pi D_5 = (3-4)(5-3)(5-4)(2-5)(2-3)(2-4)(1-2)(1-5)(1-3)(1-4)$;

Section 8.6.

1. The second has weight 192;

2(a). g of part (a) has weight x^2y^2;

4. $2a^3b + 2ab^3 + a^2b^2$;

8. 3280;

16. 11;

19(e). $1 + x$;

20(c). if the entries of (x_1, x_2, x_3) give the colors of a, b, c, respectively, then equivalence classes are $\{(0,0,0)\}$, $\{(0,0,1)\}$, $\{(0,1,0),(1,0,0)\}$, $\{(0,1,1),(1,0,1)\}$, $\{(1,1,0)\}$, $\{(1,1,1)\}$.