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Optimal US Coast Guard Boat Allocations with Sharing

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Abstract

The United States Coast Guard (USCG) presently allocates resources, such as small boats, to fixed locations (stations) for periods of one year or longer. We present a model that allows the USCG to assess novel boat allocations in which stations may "share" boats, thus cutting down on the total number of boats required. A shared boat is assigned to one particular station for each sub-portion of the year. A key innovation of the analysis is to characterize the problem in terms of "sharing paths," rather than modeling individual boats. The model uses Mixed Integer Programming to capture a subtle set of constraints and business rules such as boat and mission requirements for individual stations, preferred amount of usage (mission hours) for the boats, and limitations on how much sharing should be allowed. The model finds a boat assignment plan (with sharing) that can minimize either the number of boats or the total cost, subject to these various constraints. The scale of operations that is meaningful to the USCG permits adequate solutions to be found on a large laptop computer.

Keywords
Resource allocation, USCG, boat sharing, optimization

1. Introduction

For centuries, the United States Coast Guard (USCG) has been charged with monitoring and protecting the United States coast line. To achieve this mission, the USCG has stations scattered across the coast lines of the U.S. where USCG boats are stationed. The USCG presently allocates resources, such as boats, to stations for periods of one year or longer. The boat allocations to USCG boat stations are dictated by various constraints and business rules. For example, there is an upper bound on how many mission hours a single boat can perform annually. In some cases, this may lead to allocations where boats are under-utilized at particular stations, yet the removal of a boat would lead to a shortage of available boat hours at the station over the course of a year. This is further complicated by the fact that boat mission hour demands show different seasonality trends at different stations.

In order to improve its use of resources, the USCG has been open to considering the sharing of boats between stations. The term "sharing" a boat refers to two or more stations using the same boat on a set schedule where it rotates between stations. If such a boat sharing method were possible, the USCG would be able to maintain the same level of responsiveness with fewer overall boats (and hence a lower associated cost).

In this article, we present the optimization model that was created to allow the USCG to investigate potential boat sharing plans. The close collaboration between a team of university researchers and a USCG team ensured that the model accurately portrayed USCG business rules and constraints. This model allows the USCG to perform "what-if" experiments and weigh the potential savings and trade-offs produced by different boat sharing assignments.

Due to practical considerations, this model is intended to consider sharing between stations that are within the same USCG sector (i.e., a group of stations that are reasonably close in geographic proximity). For example, it would be impractical for a station in the New England area to share a boat with a station in Southern California. However, the model is general enough to consider any set of USCG boat stations. Additionally, the model is intended to consider only a single boat type, in particular the USCG Response Boat-Small (RB-S). The RB-S is the smallest boat in the USCG boat fleet and also the easiest to move from one station to another.

The notion of boat sharing considered for the model is defined as follows. The USCG user will specify several (not necessarily equal) time periods during the year; this will be a partitioning of the months in a calendar or fiscal year. For example, the user may specify that there are three time periods: (1) October to January, (2) February to May, and (3) June to September. Each boat will be assigned to exactly one station during each time period. Then, boats may change assigned stations at the end of each time period but never during the time period. A boat is shared between two stations if it is assigned to one station during a time period and assigned to another station during a different time period.

Given these boat sharing rules along with additional user inputs/constraints, the implemented version of this model will have two possible objectives: minimize the number of boats or minimize the cost. The model will consider pre-generated *feasible boat-paths* (i.e., possible sequences of station assignments for a boat through the year) and will pick some choice of these boat paths that satisfies the constraints and minimizes the objective function (either number of boats or total cost). It is important to note that one conceptual novelty in our method is that we model boat-paths instead of individual boats in our optimization approach. The model was formulated as a mixed integer linear programming problem and implemented in the optimization suite FICO Xpress.

Our contributions are as follows:

- A mathematical model for optimal boat allocations with sharing between USCG stations
- The modeling of boat-paths (instead of individual boats) in the optimization
- Some preliminary analysis showing that boat sharing could reduce the number of boats needed while satisfying USCG mission demands

The rest of the article is outlined as follows. First, some additional background and prior work will be discussed in Section 2. Next, the necessary parameters and decision variables for the model will be described in Section 3. The notion of feasible boat-paths will also be more rigorously defined. In Section 4, the mathematical model (objective functions and constraints) will be defined and discussed in greater detail. The software implementation will be described in Section 5, and some preliminary results and future work will be shared in Section 6. Finally, conclusions will be presented in Section 7.

2. Background and Prior Work

In prior work, researchers at the Command, Control, and Interoperability Center for Advanced Data Analysis (CCICADA), Rutgers University have collaborated with the USCG on a related boat allocation project named Boat Allocation Module (BAM I). BAM I [1–3] sought to create a mathematical model which could produce optimal assignments of small boats across the USCG boat stations so that all station requirements (e.g., mission demands) were met. The focus of this paper is the second phase of this project, known as Boat Allocation Module II (BAM II).

These models fall under the Coastal Operation Analytical Suite of Tools (COAST) for the USCG and are part of an "Engage to Excel" (E2E) university initiative funded by the Department of Homeland Security (DHS). Under the E2E program, DHS university centers of excellence (COE) work very closely with DHS agencies to accelerate the development of cutting edge solutions to real operational problems by collaborating from the beginning of the problem formulation to transition of complete pieces. COAST is a set of modules that are usable individually but rationally linked, and BAM I was one of the earliest models developed for COAST.

2.1 Boat Allocation Module I

For the boat allocation problems, the USCG boat stations are organized into geographic districts. Each district is made up of sectors, which manage individual boat stations. Every boat station has a number of requirements which must be satisfied, such as having a minimum number of total boat hours set aside for use toward each mission, as well as enough boats to cover the station's designated capabilities. A capability of a boat is the technical ability of carrying out a certain type of activity. Different boat types may have different sets of capabilities, and each station has a required set of capabilities that have to be covered by the capabilities of the assigned boats at the prescribed capacity level.

BAM I was modeled as a variant to the well-studied Resource Allocation Problem, an optimization problem found throughout the Operations Research literature. BAM I was modeled as a mixed-integer-program (MIP) with binary, integer, and continuous variables. BAM I limited the budget as a constraint and optimized a measure of unmet task "demand," which was formalized as part of the project. BAM I did not focus on individual boat assignments, but rather the resource was the total number of boats of a type at a given station. This helped to reduce the size of the problem, and it also gave the analyst and commanding officers flexibility when considering boats individually. The total number of hours allocated for use across all assigned boats of a type, per station, per mission type, was a second type of resource. BAM I did not consider the assignment of personnel to stations or boats. It was assumed that once boats were assigned to stations, and mission hours to boats, only then will personnel be appropriately stationed by commanding officers. The problem is then restated: to find an assignment of boats to stations and mission hours to such boats that satisfies all USCG business rules so that a weighted total of all stations' mission hours gone unmet is minimal. The weights of each mission, per station, represent the importance of some over others. Business rules include covering the capability requirements at each station, scheduling hours to assigned boats within a specified range determined by maintenance requirements, allotting enough hours per boat type for adequate training of personnel, and spending no more of the budget than allowed.

Allowing fractional solutions in BAM I led to the idea of sharing boats between stations in BAM II. The "sharing" of resources in the application of boats and boat stations is the concept that a boat may move between stations over the course of time (e.g. a year), shifting between various stations or else staying stationary at a given station (and not being "shared") during each time period. Even when allowing for transfer costs to switch boats between stations, this potentially lessens the number of boats required to fulfill USCG business rules and missions and can be cost-saving. BAM I was able to show that "sharing" of boats could reduce the number of required USCG boats but did not show exactly how to implement this. Let us note that while the fractional solution to BAM I provides a hint that sharing may save resources, itself however does not give enough information of which boats should be shared between which stations (and between how many), and of how to schedule the sharing of a single boat throughout the year. The work discussed in this paper (BAM II) provides a model for finding "good" boat sharing plans. It is important to note that sharing boats introduces many new (and subtle) business rules and constraints that had to be accurately captured in this new optimization model.

2.2 Related Work

Researchers at CCICADA also worked with the USCG to develop models for finding efficient allocations for their aircraft to USCG air stations. That project, named Aviation Capability and Capacity Assignment Module (ACCAM) [4], is conceptually quite different from BAM I since there are a completely different set of USCG business rules associated with their aircraft. An earlier phase of ACCAM, known as ACCAM-Simulation [5], involved simulating aviation mission readiness at individual USCG air stations. Both ACCAM projects are substantially different from BAM II since neither attempts to optimize over resource sharing plans.

There are many related examples of resource allocation problems, which seek to find optimal allocations of a finite resource to specific activities that minimize/maximize an objective value subject to some constraints [6]. One classic example is utilizing resources to effectively fight forest fires [7]. This approach uses a dynamic model to continually re-allocate resources as the fire spreads, which is conceptually different from our work in BAM II. In our work, we find an optimal resource sharing plan for a one year period, covering forecasted (regular) mission demands, and not focusing on arising emergencies.

Another example of a resource allocation problem is planning for an emergency response to a public health crisis [8]. This problem determines locations for dispensing aid such as vaccines or drugs and then allocating resources to the chosen locations. BAM II, in contrast, has pre-determined locations (boat stations) to which resources (boats) must be allocated. A similar paper studies resource allocation with a large fleet of buses [9], however their work does not consider resource sharing in a way that is applicable to BAM II.

There are a few other examples in the literature of modeling projects with the USCG. For example, Purdue University [10] developed a visual system to analyze historical USCG response and assess risks that may occur in the surrounding maritime environment. It analyzed patterns, trends, and anomalies among USCG Search and Rescue (SAR) operations, allowing the USCG to identify areas with elevated risk. Since this focuses on SAR missions, this differs in objective with BAM II. Another example is a qualitative study [11], written in the 1970's, describing resource allocation and the USCG but more from a political/social science perspective. This study does not attempt to create a mathematical model, as we do here.

3. Definition of Parameters and Variables

First, we assume that there are S stations and that the stations are labeled $1,\dots,S$. The set of stations will be denoted by $\mathcal{S} := \{1,\dots,S\}$. Additionally, we assume that there are T time periods and that the time periods are labeled $1,\dots,T$. The set of time periods will be denoted by $\mathcal{T} := \{1,\dots,T\}$. For each $i \in \mathcal{T}$, we define τ_i to be the duration (in months) of time period i. Note that $\tau_1 + \ldots + \tau_T = 12$. It will be clear from context whether we are referring to a specific station or specific time period.

3.1 Parameters for the Mathematical Model

We have the following parameters for our model. The parameters in Table 1 will be provided by a USCG user through a software user interface (later described in Section 5).

Table 1: User input parameters (from user interface)

Type	Notation	Description
Set	$\mathcal{T}$	Set of time periods $\{1,\dots,T\}$
Binary	δ_b	Value is 1 if boat sharing is allowed; 0 if not
	δ_w	Value is 1 if number of boats shared should be bounded; 0 if not
	δ_α	Value is 1 if cost of individual boat moves should be bounded; 0 if not (only used when minimizing cost)
Constant	A	Maximum number of hours each boat can work in a year
	A_L	Lower bound on preferred range each boat works in a year
	A_U	Upper bound on preferred range each boat works in a year
	μ	Maximum number of boat moves that each boat is allowed per year
	σ	Maximum number of different stations that each boat can be assigned to
	η	Minimum percentage of the total annual hours that each boat must work at its home station
	Γ	Maximum percentage of total annual hours that each boat is allowed to run in any individual time period
	γ	Minimum percentage of total annual hours that each boat must run in any individual time period
	ϕ	Minimum percentage of total annual hours that each boat must run
	ε	Extra percentage of the max annual hours that each individual boat may work as "overtime"
	ε_g	Average extra percentage of the max annual hours that the boats may work as "overtime"
	ε_u	Penalty factor for unbalanced boat hour distributions
	θ	Cumulative boat hour overage allowed through a station
	C	Fixed cost for each boat (only used when minimizing for cost)
	f	Small fixed cost/penalty for each boat move (only used when minimizing number of boats)
	w	Maximum number of boats that are shared throughout the region (only used if $\delta_w = 1$)
	α	Maximum cost allowed per boat move (only used if $\delta_\alpha = 1$)

Additional input parameters will be provided by the user through an Excel data input file (later described in Section 5). Those parameters are described in Table 2.

3.2 Generating Feasible Boat-paths

In addition to defining the user input parameters, it is necessary to introduce the notion of *feasible boat-paths*. We first define a *station assignment* to be any function $\pi : \mathcal{T} \to \mathcal{S}$. Each such function represents a possible way that a boat may be assigned to various stations through the year (e.g., $\pi(1)$ is the station for the first period, $\pi(2)$ is the station

Table 2: User input parameters (from Excel input file)

Type	Notation	Description
Set	$\mathcal{S}$	Set of stations $\{1,\ldots,S\}$
Array	$h_{s,t}$	The number of boat hours needed at station s during time period t (for each $s \in S$ and $t \in T$)
	$d_{s,s'}$	The cost for moving a single boat from station s to station s' (for each $s,s' \in S$); these values will vary when minimizing cost, but these values will all be f if minimizing number of boats
	$e_{s,s'}$	Either 0 or 1; the value is 1 if a boat is allowed to move from station s to station s', and 0 if not (for each $s,s' \in S$)
	$r_{s,s'}$	Monthly cost for a boat to transfer from its home station (s) to a different station (s'). (for each $s,s' \in S$)
	L_s	The minimum number of boats that must be assigned to station s during each time period (for each $s \in S$)
	U_s	The maximum number of boats allowed at station s during each time period (for each $s \in S$)
	M_s	The maximum number of total boat moves that station s can be involved in throughout the year (for each $s \in S$)

for the second period, etc.). In addition, a home station s may be chosen out of $\pi(\mathcal{T}) := \{\pi(t) \,:\, 1 \le t \le T\}$ (the set of stations that appear as an assignment). For each such function π, we can compute the cost of all the boat moves needed to realize this station assignment by

$$c := d_{\pi(T),\pi(1)} + \sum_{t=1}^{T-1} d_{\pi(t),\pi(t+1)} + \sum_{i=1}^{T} \tau_i \cdot r_{s,\pi(i)}, \qquad \text{(BPcost)}$$

where $d_{s,s'}$ is the cost of moving a single boat from station s to station s' and $r_{s,s'}$ is the monthly cost for a single boat to be transferred from (home) station s to a different station s'. The tuple (π,c,s) is a *boat-path* if it satisfies the above requirements. It should be re-emphasized that given a choice of the function π and a home station s (limited to stations that occur in π), the cost c is determined.

We define the *feasible boat-paths* as the set of boat-paths which satisfy additional constraints. If no boat sharing is allowed ($\delta_b = 0$), then the set of feasible station assignment functions would be $\{\pi \,:\, \pi(i) = s, 1 \le i \le T\}$ for each fixed s. Since there are no boat moves in each of these functions, the cost c would be 0. Our set of feasible boat-paths would then be

$$\Pi := \{(\pi,0,s) \,:\, s \in \mathcal{S},\, \pi(t) = s \;\forall t \in \mathcal{T}\}.$$

Note that there are exactly S feasible boat-paths, and each boat-path corresponds to a boat being assigned to a particular station for the whole year.

On the other hand, suppose that boat sharing is allowed ($\delta_b = 1$). For each possible home station $s \in \pi(\mathcal{T})$, we have the cost of this assignment given by Eq. (BPcost) and a potential boat-path (π,c,s). Then, a station assignment function π is feasible if it satisfies the following conditions:

(1) *All transfers allowed*: for each $1 \le i \le T-1$, $e_{\pi(i),\pi(i+1)} = 1$ and $e_{\pi(T),\pi(1)} = 1$

(2) *Max boat moves*: the number of times $\pi(i)$ differs from $\pi(i+1)$ for $1 \le i \le T-1$ (and 'plus 1' if $\pi(T)$ differs from $\pi(1)$) must be at most μ

(3) *Max stations visited*: the number of elements in the set $\{\pi(i) : 1 \le i \le T\}$ must be at most σ

(4) *Max cost per boat move*: if $\delta_\alpha = 1$, then c must be at most α

Our set of feasible boat-paths would then be

$$\Pi := \{(\pi,c,s) \,:\, \pi \text{ is feasible},\ c \text{ satisfies Eq. (BPcost)},\ s \in \pi(\mathcal{T})\}.$$

We will denote the number of feasible boat-paths by $P := |\Pi|$. We will consider the boat-paths to be labeled by $1,\ldots,P$ and will refer to the p-th feasible boat-path as (π_p, c_p, s_p). By convention, we will also assume that $1 \le p \le S$ corresponds to the "constant paths" (paths where boats do not move). In particular, we assume that (π_1,c_1,s_1) corresponds to the path where a boat is assigned to station 1 for the whole year, (π_2,c_2,s_2) corresponds to the path where a boat is assigned to station 2 for the whole year, and so on.

3.3 Decision Variables for the Mathematical Model

Now, we have the following decision variables for our model, as shown in Table 3.

Table 3: Model decision variables

Type	Notation	Description
Integer	y_p	Total number of boats with the boat-path (π_p, c_p, s_p)
Continuous	$x_{p,t}$	Total number of hours that all boats on boat-path (π_p, c_p, s_p) work during time period t
	z_p^1	Total under-utilization (below A_L hours per boat) of all boats on boat-path (π_p, c_p, s_p)
	z_p^2	Total over-utilization (above A_U hours per boat) of all boats on boat-path (π_p, c_p, s_p)

It is important to emphasize that a boat-path merely corresponds to a "route" that a boat can be assigned. If two boats followed boat-path p, we view this as a single instance of the path p with its weight $y_p = 2$.

4. Mathematical Model

We finally present the mathematical model. First, we present the two possible objective functions to minimize.

The first possible objective is to minimize the number of boats necessary. The objective function for this is

$$BOATS := \sum_{(\pi_p, c_p, s_p) \in \Pi} (1 + c_p) y_p + \varepsilon_u \sum_{p=1}^{P} (z_p^1 + z_p^2). \tag{ObjBoats}$$

Also, note that all costs c_p will be based on the assumption that each boat move costs the fixed penalty f. This allows us to minimize the number of boats but also to reduce unnecessary boat moves. In addition, the z_p^1 and z_p^2 terms enforce a penalty on unbalanced boat workloads (outside of the range of A_L and A_U assigned annual hours). Note that going outside of the range for a cumulative total of $1/\varepsilon_u$ hours would incur a cost equivalent to one boat.

The second possible objective is to minimize the total cost. The objective function for this is

$$COST := \sum_{(\pi_p, c_p, s_p) \in \Pi} (C + c_p) y_p + \varepsilon_u C \sum_{p=1}^{P} (z_p^1 + z_p^2). \tag{ObjCost}$$

Note that in this case, all costs c_p are based on boat move costs that the user supplied (where generally, the costs of boat moves varies depending on the stations involved). Again, the z_p^1 and z_p^2 terms enforce a penalty on unbalanced boat workloads, and going outside of the range by a total of $1/\varepsilon_u$ hours would incur a cost equivalent to one boat (in this case, C).

For notational convenience, we define the function Δ on stations s_1, s_2, s_3 by

$$\Delta(s_1, s_2, s_3) := \begin{cases} 1 & \text{if } s_1 \neq s_2 \text{ and } s_3 \in \{s_1, s_2\} \\ 0 & \text{otherwise.} \end{cases} \tag{MoveCheck}$$

4.1 Objective Function and Constraints

The objective function(s) and constraints are summarized in the following MIP.

$$\text{Minimize } BOATS \quad \text{or} \quad \text{Minimize } COST \tag{ObjFunc}$$

subject to

$$\sum_{t=1}^{T} x_{p,t} \leq A(1+\varepsilon) y_p \qquad 1 \leq p \leq P \tag{1}$$

$$\sum_{p=1}^{P} \sum_{t=1}^{T} x_{p,t} \leq A(1+\varepsilon_g) \sum_{p=1}^{P} y_p \tag{2}$$

$$\sum_{\substack{1 \leq t \leq T \\ \pi_p(t) = s_p}} x_{p,t} \geq \eta \sum_{t=1}^{T} x_{p,t} \qquad 1 \leq p \leq P \tag{3}$$

$$x_{p,t} \le \Gamma A y_p \qquad 1 \le p \le P,\ 1 \le t \le T \tag{4}$$

$$x_{p,t} \ge \gamma A y_p \qquad 1 \le p \le P,\ 1 \le t \le T \tag{5}$$

$$\sum_{t=1}^{T} x_{p,t} \ge \phi A y_p \qquad 1 \le p \le P \tag{6}$$

$$\sum_{\substack{1 \le p \le P \\ \pi_p(t)=s}} x_{p,t} \ge h_{s,t} \qquad 1 \le s \le S,\ 1 \le t \le T \tag{7}$$

$$\sum_{\substack{1 \le p \le P \\ \pi_p(t)=s}} y_p \ge L_s \qquad 1 \le s \le S,\ 1 \le t \le T \tag{8}$$

$$\sum_{\substack{1 \le p \le P \\ \pi_p(t)=s}} y_p \le U_s \qquad 1 \le s \le S,\ 1 \le t \le T \tag{9}$$

$$\sum_{1 \le p \le P} \sum_{1 \le t \le T-1} \Delta(\pi_p(t), \pi_p(t+1), s) y_p + \sum_{1 \le p \le P} \Delta(\pi_p(T), \pi_p(1), s) y_p \le M_s \qquad 1 \le s \le S \tag{10}$$

$$x_{p,t} \ge 0 \qquad 1 \le p \le P,\ 1 \le t \le T \tag{11}$$

$$y_p \in \mathbb{Z}_{\ge 0} \qquad 1 \le p \le P \tag{12}$$

$$\sum_{\substack{1 \le p \le P \\ \pi_p(t)=s}} \sum_{q=1}^{T} x_{p,q} \le \theta + A \sum_{\substack{1 \le p \le P \\ \pi_p(t)=s}} y_p \qquad 1 \le s \le S,\ 1 \le t \le T \tag{13}$$

$$z_p^1, z_p^2 \ge 0 \qquad 1 \le p \le P \tag{14}$$

$$A_L \cdot y_p - z_p^1 \le \sum_{q=1}^{T} x_{p,q} \le A_U \cdot y_p + z_p^2 \qquad 1 \le p \le P \tag{15}$$

and if $\delta_w = 1$,

$$\sum_{S+1 \le p \le P} y_p \le w \tag{16}$$

We briefly summarize the roles of these constraints:

(1) guarantees that each boat works at most $A(1+\varepsilon)$ hours annually;

(2) guarantees that the average amount the boats work is at most $A(1+\varepsilon_g)$ hours annually;

(3) guarantees that each boat works at its home station for at least ηH hours, where H represents the total number of hours the boat worked;

(4) guarantees that each boat works at most ΓA hours in any given time period;

(5) guarantees that each boat works at least γA hours in any given time period;

(6) guarantees that each boat works at least ϕA hours annually;

(7) guarantees that the required boat hours $h_{s,t}$ at station s during time period t are covered for each s and t;

(8) guarantees that the number of boats at station s during time period t is at least L_s for each s and t;

(9) guarantees that the number of boats at station s during time period t is at most U_s for each s and t;

(10) guarantees that the number of total boat moves that station s is involved in is at most M_s for each s;

(11) guarantees that the number of hours each boat works during any given time period is non-negative;

(12) guarantees that the number of boats assigned to boat-path p is a non-negative integer for each p;

(13) guarantees that the set of boats at station s during time period t do not jointly exceed θ hours beyond their preferred maximum of A hours annually for each s and t;

(14) guarantees that the under/over-utilization of boats are non-negative;

(15) guarantees that the variables z_p^1 and z_p^2 measure boat under-utilization below A_L and over-utilization above A_U, respectively;

(16) guarantees that no more than w boats are shared throughout the system considered (only if the user requests an upper bound).

5. Implementation

This model was implemented in the optimization suite FICO Xpress (Mosel) so that the USCG can easily run the optimization using different input parameters. We describe some of the features of the software implementation below.

5.1 Optimization Solver Features

It is likely that there are multiple solutions providing the optimal value (or a near-optimal value). Because of this, the Xpress implementation allows for the USCG user to request the "N best solutions" for any value of N. This allows for the user to compare multiple solutions given the same set of input parameters.

In addition, the user may enforce an optional maximum run time for the solver. This forces the solver to return the best solution(s) it has found until that point. In practice, the solver usually finds the optimal (or a near-optimal) solution very quickly, but without enforcing a stop time, the solver will continue running until it proves the solution's optimality. For example, a stop time of even 5-10 minutes often produces a near-optimal solution, but without a stop time, the solver may run for hours on the same set of input parameters.

5.2 Parameter Inputs

Some of the input parameters, namely the parameter arrays listed in Table 2, are contained within an input data Excel spreadsheet. The remaining parameters are entered using a graphical user interface that was developed in Xpress (see Figure 1). This allowed the USCG user to easily change parameters and perform "what-if" experiments on potential savings and trade-offs.

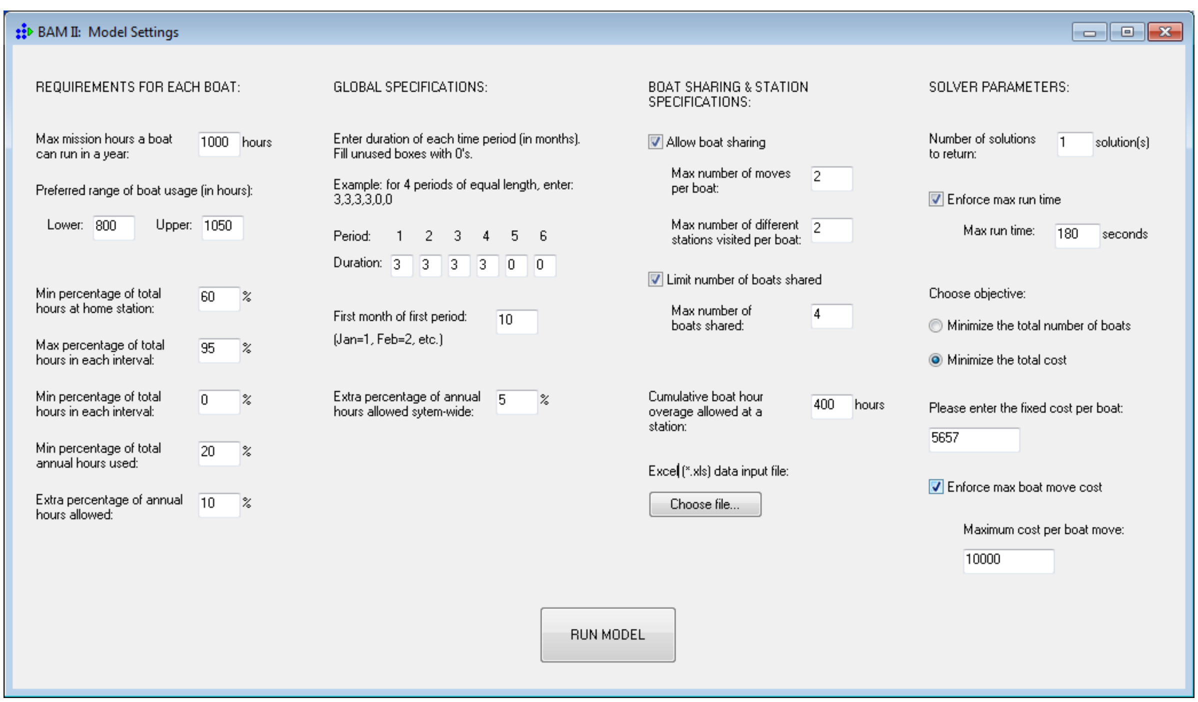

Figure 1: BAM II user interface

5.3 Graphical Output

Since a lengthy text output alone would be difficult to interpret, a graphical output was also developed. However, one difficulty was in finding an effective visual output that contained the most important information without being visually overwhelming. The resulting graphical output is shown below (see Figure 2). Note that in the interest of page space, we only give a small example using notional data. The model can easily run for sectors with over 10 stations.

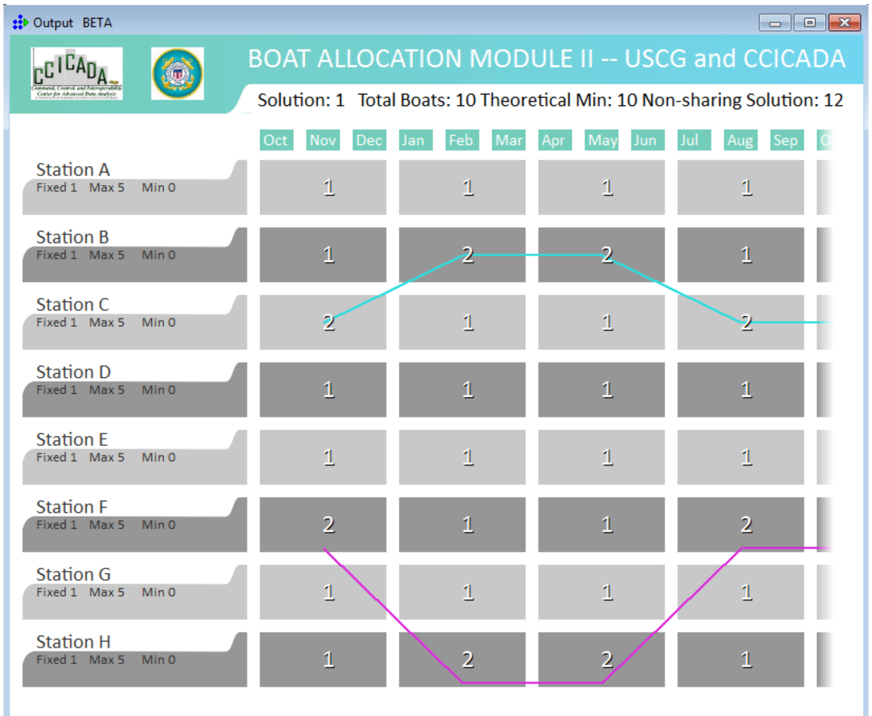

Figure 2: Graphical visual output

In the visual output, stations are given in the rows and time periods are given in the columns. The lines represent non-stationary boat-paths. To reduce visual clutter, the constant boat-paths (with no boat moves) are not drawn. The number in each box represents the number of boats at that particular station during that particular time period.

In addition, each optimization run of the software produces an output HTML document that archives the run's input parameters, the text output providing the specific boat sharing plan, and the corresponding graphical output. This allows for easier comparison between multiple runs with different parameters.

6. Preliminary Results and Future Work

The USCG has been pleased with the development of this model and the new capabilities it provides for investigating the potential benefits of boat sharing. This work is expected to be implemented using real data for various USCG sectors and boat stations. Using unofficial approximated data provided by our USCG partners, it seems very possible for some sectors to meet mission demands with 1-2 fewer boats by sharing (when compared with optimal non-sharing solutions). Maintaining boats at an operational level takes many man-hours of work each year. This model has the potential to free up personnel from some of that maintenance to conduct other vital Coast Guard missions.

In future work, we may consider implementing more sophisticated analyses for the cost of boat moves, designating certain number of boats to be stationary at certain stations (i.e., not allowed to move), and restricting boat moves between time periods (e.g., enforce a maximum for the number of boat moves between two periods or restrict some combinations of boat moves from happening simultaneously). Furthermore, this optimization model could be useful in other resource allocation problems both for the USCG and in other settings. This model could easily be adapted for other resource sharing problems. We intend to investigate such further applications in the future.

7. Conclusion

Because of the close collaboration between a university team and a USCG team, we were able to develop a model that produced optimal boat sharing plans that captured important USCG business rules and constraints for boat allocations. The resulting Xpress software has a user interface and produces visualizations of the optimal solutions, allowing the user to run "what-if" experiments with different parameters and easily compare solutions.

This problem also allowed us to develop some novel modeling approaches using boat-paths instead of modeling individual boats. We expect some of the methods used to be applicable in similar resource allocation problems where sharing is allowed.

8. Acknowledgements

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