Mathematics of Planet Earth: Management of Natural Resources

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Mathematics of Planet Earth 2013

- A joint effort initiated by North American Math Institutes: **MPE2013**
- More than 100 partner institutes, societies, and organizations in UK, France, South Africa, Japan, and all over the world
- www.mpe2013.org

Mathematics of Planet Earth 2013

- Activities world-wide throughout 2013
- Sponsorship by UNESCO
- Support from Simons Foundation
- Workshops, tutorials, competitions, distinguished lectures, educational programs

Mathematics of Planet Earth Beyond 2013

- Problems of the planet will not go away in one year.
- We are organizing a series of events to continue beyond 2013.
- New initiative world-wide now called **MPE**
- In the US, we call it **MPE2013+**
- NSF support

Mathematics of Planet Earth Beyond 2013

Goals of MPE2013+

- Involve mathematical scientists in addressing the problems of the planet
- Enhance collaborations between mathematical scientists and other scientists
- Involve students and junior researchers in the effort
- Encourage life-long commitment to working between disciplines to solve the problems of society

Mathematics of Planet Earth 2013+

- Opening Introduction to Problems of the Planet and involve students and junior faculty: Arizona State University, Jan. 7-10, 2014
- Five ***Research Clusters***, beginning with workshops:
  - ***Sustainable Human Environments*** (Rutgers U.), April 23-25, 2014
  - ***Global Change*** (UC Berkeley), May 19-21, 2014
  - ***Data-aware Energy Use*** (UC San Diego), Sept. 29 – Oct. 1, 2014
  - ***Natural Disasters*** (GA Tech), May 13-15, 2015
  - ***Management of Natural Resources*** (Howard University), June 4-6, 2015

Mathematics of Planet Earth 2013+

Follow-up cluster activities:

- Sustainable Human Environments cluster:
  - Pre-workshop: Urban Planning for Climate Events Sept. 2014; Post-workshop: Tentatively in Paris, Fall 2015
  - Cluster activities of various kinds
- Natural Disasters cluster: working with several potential partners in Mexico and Colombia.
- Global Change cluster: considering a follow-up event at the National Center for Atmospheric Research (NCAR) and one at Old Dominion U. on communication of global change challenges

Mathematics of Planet Earth 2013+

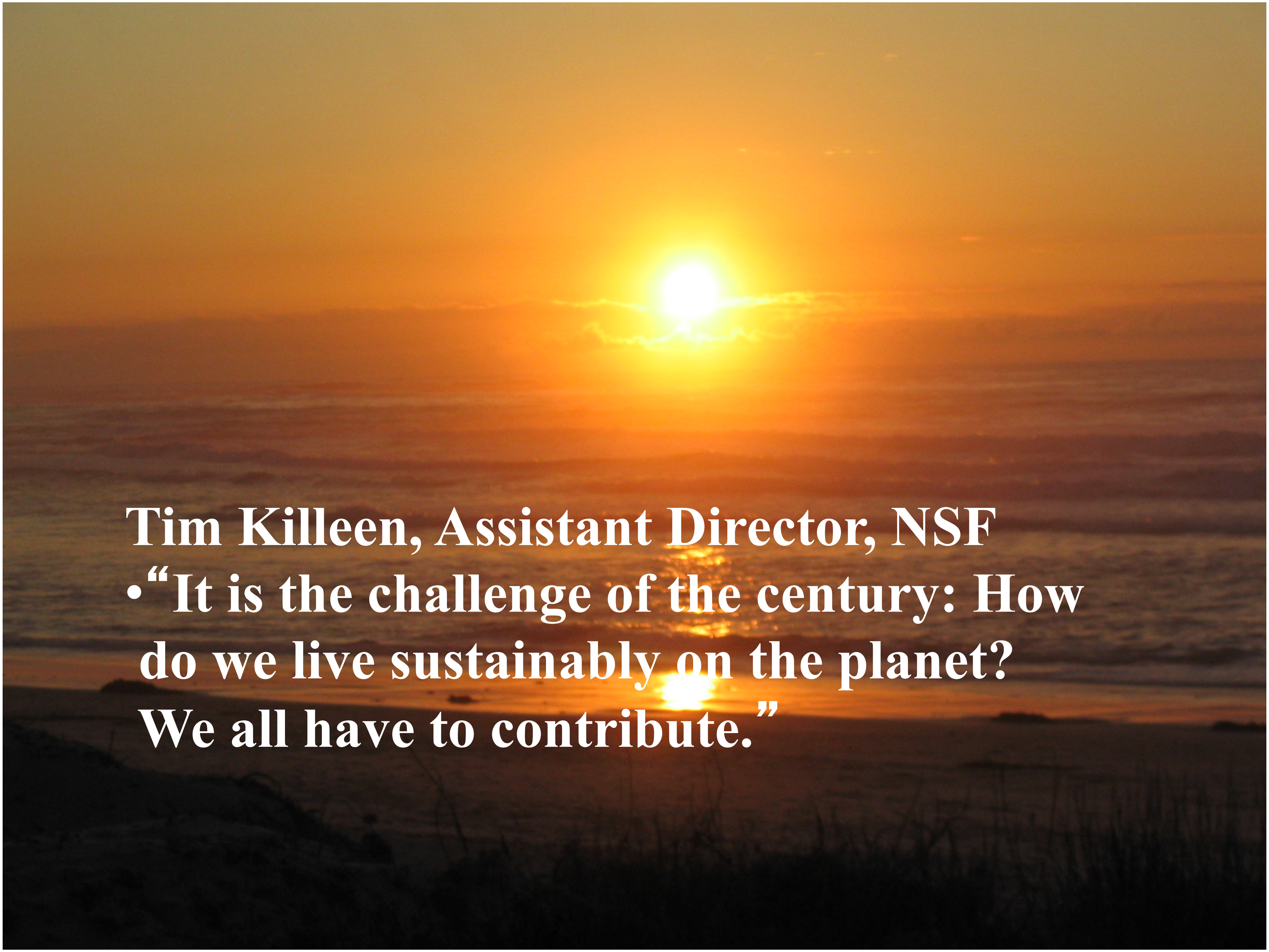
Follow-up cluster activities:

- Management of Natural Resources cluster and Global Change cluster:
  - Looking into the possibility of follow up in Africa
- All clusters:
  - Looking into possibility of research groups ("squares") at American Inst. of Mathematics (AIM)

Mathematics of Planet Earth 2013+

- Education is a crucial piece of this and of the sustainability effort
  - Workforce development
  - Public literacy
- Need education at all levels, starting with K-12.

- Education issues in each workshop
- Special Education cluster: ***Education for the Planet Earth of Tomorrow***
- Cluster workshop: U. of Tennessee, Sept. 30 – Oct. 2, 2015.


Tim Killeen, Assistant Director, NSF
•"It is the challenge of the century: How do we live sustainably on the planet? We all have to contribute."

Management of Natural Resources

- Workshop will be concerned with:
  - Forests
  - Water
  - Food
  - Animals, Plants, Ecosystems
- But all of the themes of MPE2013+ are relevant.
- Example: Big emphasis nowadays on the nexus of food, energy, water, and climate.
- ***"When we try to pick out anything by itself in nature, we find it hitched to everything else in the universe."*** – John Muir, 1911

The Nexus of Food, Energy, Water, and Climate

- It takes water (lots of water) to produce food and energy
- Energy is needed to treat water
- Energy is needed to produce food
- Food crops can be used as a source of energy
- Production of energy affects the climate
- Climate change affects crops
- And so on.

Management of Natural Resources

- Will discuss sample mathematical sciences challenges from food, water, forests, animals/ plants/ecosystems, but also their interconnection.
- Will describe some examples of projects I've been involved with
- All by way of introducing some key themes for the workshop

Starting Point: Global Change

- The planet is constantly changing.
- But the pace of change has accelerated as a result of human activity:
  - Construction and deforestation change habitats
  - Over-fishing reduces wild populations
  - Fossil fuel combustion leads to atmospheric greenhouse gas buildup
  - Commerce and transport introduce non-native species

Global Change

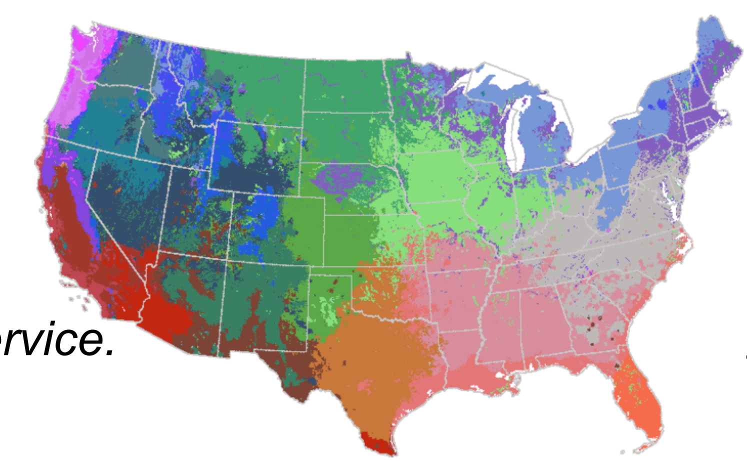
- We need to:
  - Monitor global change to understand processes leading to change
  - Learn how to mitigate and adapt to its effects
  - Determine if we are meeting goals for our planet
  - Get early warning of dangerous trends

Global Change: Data

- ***The Age of Observation***:
  - The unprecedented amount of data about health of the planet provides great opportunities but also poses immense challenges
  - How do we choose what to observe and what data to save?
  - What are appropriate sampling and monitoring designs?
  - How to reconcile so many different variables with so many different spatiotemporal characteristics?

Global Change: Data NEON

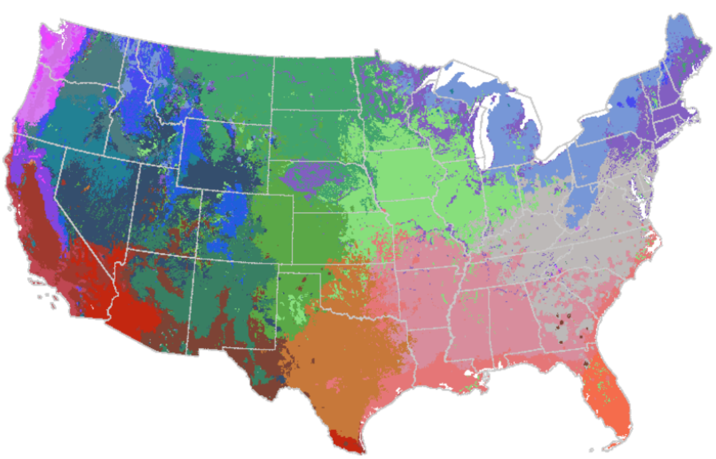
- National Ecological Observatory Network (NEON) is collecting data at 20 sites across the U.S. to get a continent-wide picture of the impacts of climate change, land use change and invasive species on natural resources, and biodiversity – all topics of interest for this workshop. Many research challenges.
- Those 20 sites needed somehow to represent the entire continent ecologically, so their selection was critical.



Credit: William Hargrove, U.S. Forest Service.

Global Change: NEON

- Those 20 sites needed somehow to represent the entire continent ecologically, so their selection was critical.
- Their first step was to divide the US into 20 ecologically homogeneous regions using methods based on firm scientific principles.



Credit: William Hargrove, U.S. Forest Service.

Global Change: NEON

- They divided the country into 8 million patches, and for each patch, they collected 9 pieces of information about its ecology and climate.
- They then used a supercomputer to cluster the patches into similar regions and picked a representative site within each region.
- They then reanalyzed the data to make sure that the 20 sites were as different from one another as possible and represented the full spread of ecological conditions.
- But it would be better to consider 100 different ecological properties rather than just 9. Computationally intractable. Challenge to develop algorithms for dealing with larger and larger data sets

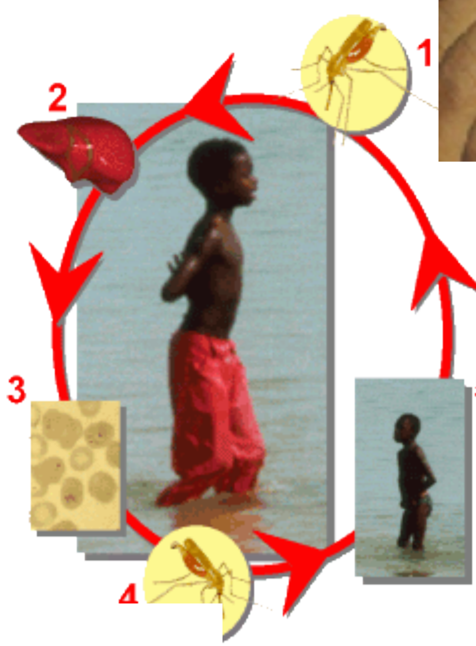
Global Change

- ***Effects of Global Change***:
  - Goal is not so much to describe the many effects of global change as to understand:
    - Interface between change in one sector on another – e.g., to understand Lyme Disease spread into Canada, need understand tick life cycles, bird migrations, climate change

    - Risk-based comparison of alternative adaptation and mitigation strategies – e.g., for control of invasive species or for more severe weather.

Global Change: Climate and Health

- Some early warning signs:
  - Malaria in the African Highlands
  - Dengue epidemics along the Rio Grande & in Brazil



Dengue Fever

Global Change: Climate and Health

- Some early warning signs:
  - Cholera affected by sea surface temperature
  - Increase in Lyme disease in Canada
  - St. Louis Encephalitis (Florida outbreak)
  - Animal and plant diseases too

–Complex interaction among climate, life cycle of hosts and vectors, migration patterns, etc.

Tick carrying Lyme Disease

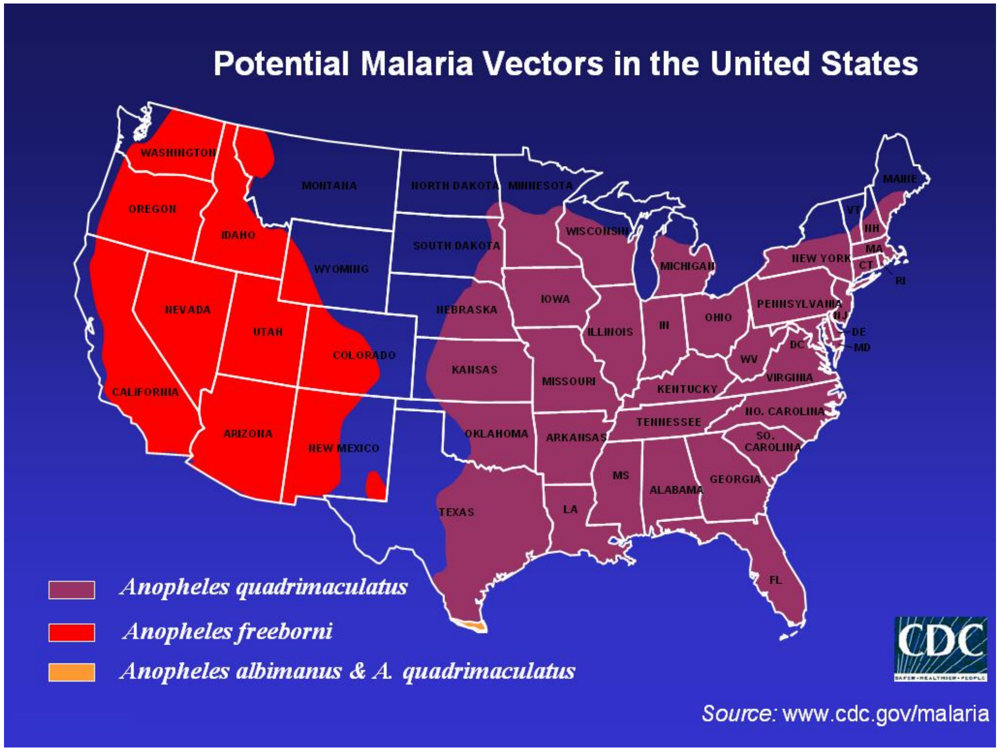
cholera

Global Change: Malaria

- The challenge of climate change: Malaria springs up in areas it wasn't in before.
- Highlands of Kenya
- Potential for Malaria in the US – Texas, Florida, Washington, …
- A project at Howard
- A key role for modelers:

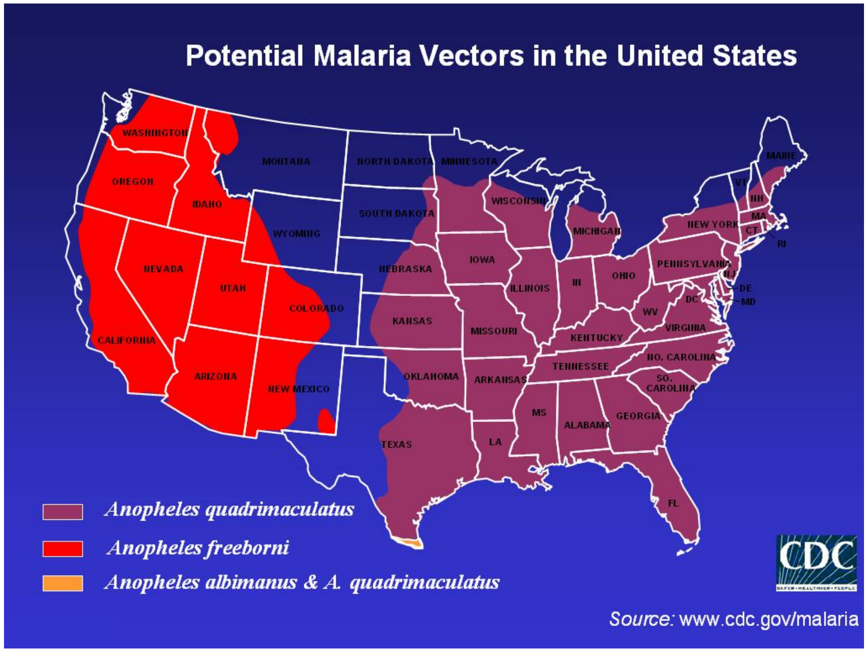
Aid in early warning:

Surveillance.



Global Change: Malaria

- Howard project:
  - Develop a mathematical model for the dynamics of malaria transmission and control in parts of Africa, taking into account both human and mosquito populations
  - Looking at effect of local malaria eradication efforts in Africa on number of malaria cases in US



Forests

- Research about environmental health often requires merging datasets produced at different times for different purposes.
- Many different government agencies and others collect data that is relevant to understanding the health of forests:
  - The Forest Service does an inventory of plots around the country every five to ten years that assesses the trees, the ground vegetation, the soils, and the air quality;
  - EPA assesses water quality around the country;
  - Private groups assess at-risk species
- Understanding the true state of our forests and the threats to them requires integrating this data coherently. A data fusion research challenge.

"Forests on the Edge"

- "Forests on the Edge" is a project that is doing just that, combining all those data sources into a single map and analyzing the results.

"Forests on the Edge"

- "Forests on the Edge" is a project that is doing just that, combining all those data sources into a single map and analyzing the results.
- But the data doesn't line up neatly.
- The plots the Forest Service analyzes are different from the plots the Geological Survey analyzes, etc.
- The data are of varying quality and are gathered in different ways. Scales vary.
- The project has developed techniques to use the combined data to produce the clearest picture of the state of our forests and the threat to it, but new techniques are needed to quantify the uncertainty of the combined data they produce. Another research challenge.

Animals, Plants, Ecosystems

- I saw this baby rhino at the Mohololo Animal Refugee Reserve in South Africa
- It was rescued from dried mud in a waterhole where its mother had perished

Animals, Plants, Ecosystems

- We were studying management of biological reserves as a DIMACS activity
- Some of the mathematical sciences issues:
  - How large should the reserve be?
  - How do you measure whether it is successful in preserving the "biodiversity"?
  - Should we treat animals for illness/injury?

Animals, Plants, Ecosystems

Animals, Plants, Ecosystems

- Some other issues:
  - Should one manage the water supply? Or let nature take its course?
  - Should one vaccinate animals?
  - Should one have taken the baby rhino from the reserve?
- Some of these can be formulated as mathematical issues.

Food

- Fish a major source of food in the world
- Management of fish populations a subject of a great deal of mathematical modeling
- Complex interactions of human and ecological systems
- Here – a slightly different perspective on management of fish populations

Fish Populations

- The United States sets rules for fishing with the goal of maintaining healthy fish populations.
- Rules depend on specific species and include
  - Allowable locations to fish
  - Allowable seasons to fish
  - Catch quotas
- Violations of the rules leads to fines – sometimes quite large

Endangered Atlantic Cod

Enforcement of Fisheries Rules

- Many agencies are involved in enforcing the fisheries rules and regulations.
- One of those agencies is the US Coast Guard.
- Through the Laboratory for Port Security at Rutgers and the CCICADA Center (the DHS center of excellence based at DIMACS) we have been working with the Coast Guard to define and enhance scoring rules to lead to better enforcement of fisheries rules.

LPS Modeling Technology Policy
LABORATORY FOR PORT SECURITY

United States Coast Guard
U.S. Department of Homeland Security

CCICADA
Command, Control, and Interoperability Center for Advanced Data Analysis
A Department of Homeland Security Center of Excellence

Enforcement of Fisheries Rules

- This work has gotten me to some interesting places.

Fisheries Law Enforcement

- The US Coast Guard District 1 (based in Boston) uses a ***scoring system called OPTIDE to determine which commercial fishing vessels to board to look for violations.***
- They asked us if their success rate in finding violations by boarding could be improved by use of sophisticated methods of data analysis.

Fisheries Law Enforcement

<u>Examples of Scoring Rule Components</u>

- Points for current or past negative intelligence reports
- Points depending upon date last boarded
- Points based on information about the type of boat
- Points for having found violations in past boardings – depending upon type of violations
- ***Board if total score (number of points) exceeds a threshold***
- Our methods did better than OPTIDE.

Fisheries Law Enforcement

- Looked at machine learning methods to see if other features, or combination of present features and new ones, can lead to decision rules that obtain higher success rate from boardings.
- Represent boarding activities by a set of features
- Aim to learn a classifier that will output "board" or "don't board" based on the features
- Choosing the features: Combination of data analysis, intuition, and a lot of trial and error

Fisheries Law Enforcement

- Our approach: boosted decision tree
- Useful for comparison to rule-based approach like OPTIDE.
- In boosting, instead of learning a single decision tree, we learn multiple decision trees on different training sets.
- We then learn the “best” weights for combining results of individual decision trees into an overall boosted decision tree

Fisheries Law Enforcement: RIPTIDE

- RIPTIDE is our classifier obtained from USCG data
- RIPTIDE = Rule Induction OPTIDE
- Our best model for RIPTIDE uses some new features, such as type of vessel (General, Trawler, Pot/Trap) and prior violations per boarding
- Much experimentation.
- ***Best model for RIPTIDE found so far outperforms OPTIDE up to 87% in an experiment***
- This model uses some features not used in OPTIDE, e.g., distance to coast, vessel subtype

Regression Models

- Looked at regression models to derive alternative weights for the same features used in OPTIDE, based on some of the data
- Developed decision rules based on derived weights
- Tested those decision rules on rest of the data
- Also improved on OPTIDE

Water

"Water, water everywhere and not a drop to drink"

- Need good quality water for drinking, agriculture, etc.
- Need ways to monitor the quality of water
- A project of DyDAN, the Center for Dynamic Data Analysis based at DIMACS, in collaboration with Sandia National Laboratories.

Source: www.cleanwateraction.org

Source: swcarr.arizona.edu

Water Quality

- Need contamination warning systems (CWS)
- Early warning of accidental or deliberate contamination of a water supply system
- Complex because water systems have many components:
  - Pipes and constructed conveyances, physical barriers, water collection, pretreatment, treatment, storage and distribution facilities

Source: www.leesburgva.gov

Source: www.thehindu.com

Water Quality

- One approach: locate water quality sensors throughout the system.
- But where?
- Sensor placement can be automated with optimization methods that computationally search for a sensor configuration that minimizes contamination risks.

Source: www.wager-quality-sensors.com

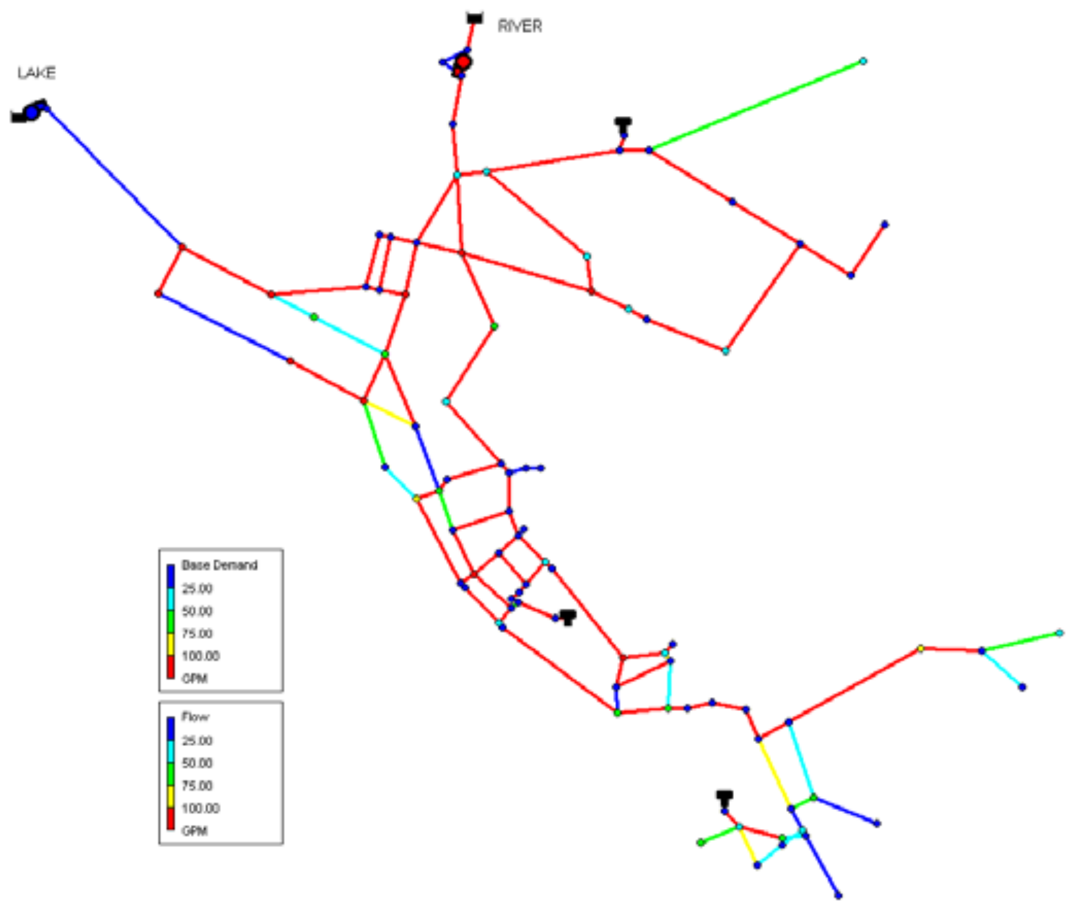
Water Quality

- Optimization methods use a computational model to estimate the performance of a sensor configuration.
- For example, a model might compute the expected impact of an ensemble of contamination incidents, given sensors placed at strategic locations.

Source: intelesense.net

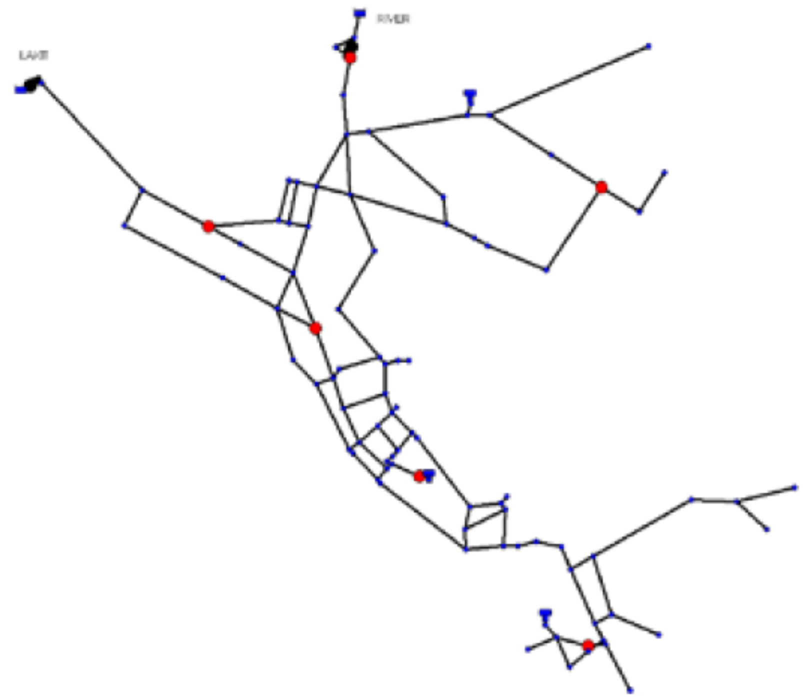
Source: www.sct-us.info

Water Quality



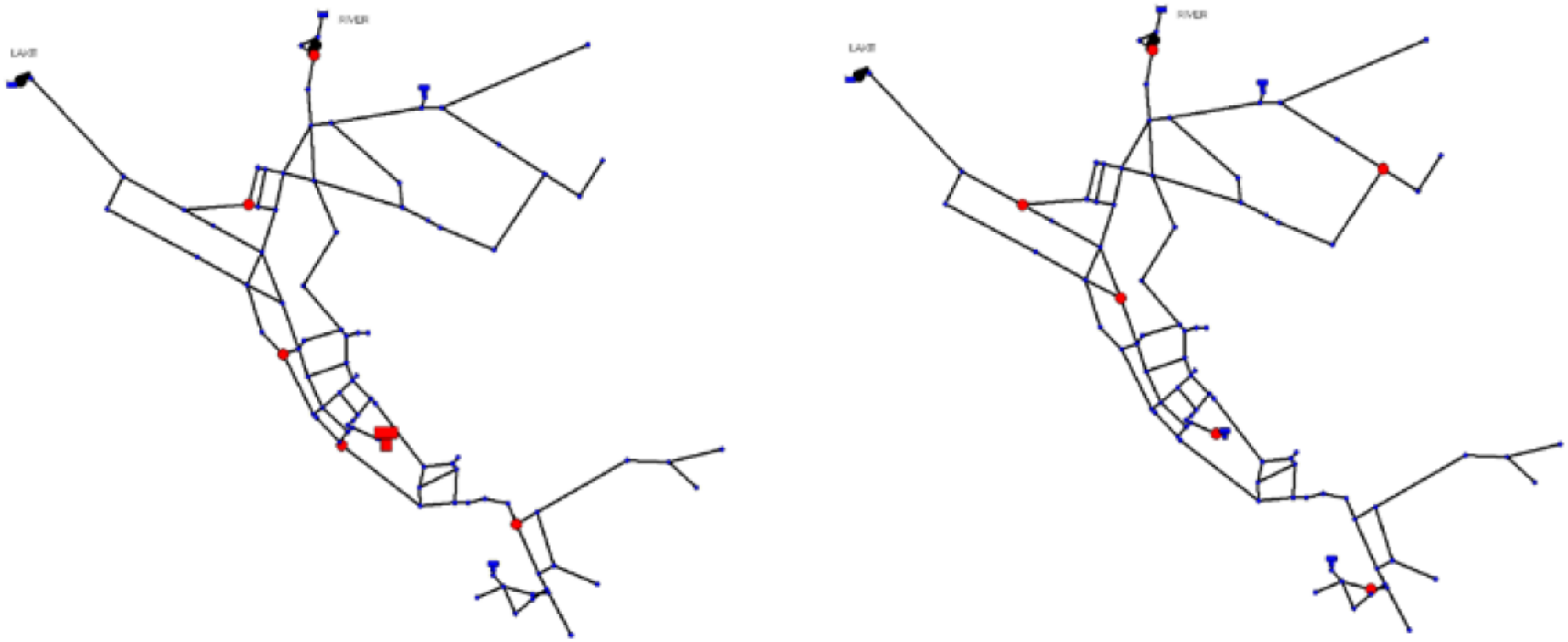
- Example of a water network served by both a river and a lake; node colors indicate base demand and pipe colors indicate bulk flow rates. Work done in collaboration with EPA
- Source: EPA, "Sensor Network Design for Drinking Water Contamination Warning Systems"

Water Quality

- Same water systems,with two examples of sensor locations with 6 sensors discussed in EPA guidance for sensor network design
- Source: EPA, “Sensor Network Design for Drinking Water Contamination Warning Systems”

Water Quality



- Two "scenarios" investigated: a chemical contamination and a biocontamination.
- Under a model, left system left fewer people sickened with a bio scenario than right system, but more with a chem scenario
- Source: EPA, "Sensor Network Design for Drinking Water Contamination Warning Systems"

Water Quality

- Different things you might want to optimize:
  - Minimize number of illnesses
  - Minimize number of failed detections (if assume some randomness in detection by sensors)
  - Minimize time of detection
  - Minimize length of contaminated pipe

Water Quality

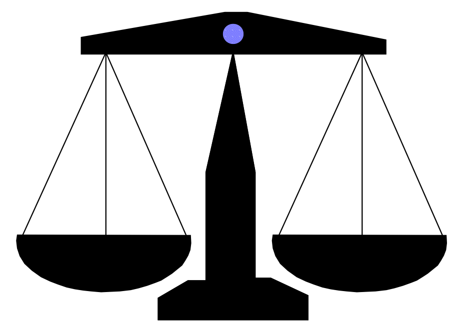
- System developed for EPA piloted at Greater Cincinnati Water Works

Source: www.wcpo.com

Water Quality

- When selecting sensor locations that minimize the mean impacts over a set of contamination incidents, this problem is equivalent to a well-known problem from the facility location literature: the ***p-median facility location problem***.
- ***p-median problem***: locate facilities so that the average distance of a “customer” to a facility is minimized

Ecosystems



Measurement of Biodiversity

•Evidence about the health of ecosystems is often obtained by measuring the "biodiversity."

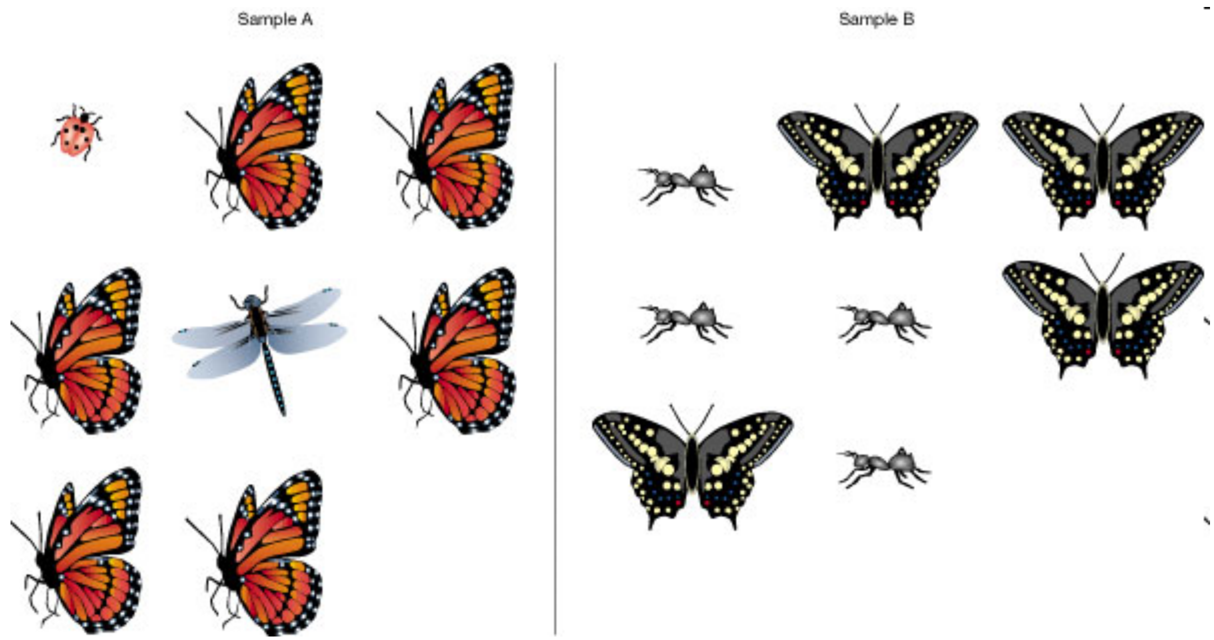
•***An index of biodiversity allows us to set specific goals and measure progress toward them.***

Dimensions of Biodiversity

- But what is biodiversity?
- 100s of papers trying to define it
- ***Biodiversity is a multidimensional concept.***
- Some components of it are:
  - –Species diversity
  - –Genetic diversity within species
  - –Ecosystem diversity
  - –Ecosystem services and processes

Measurement of Biodiversity

- Which of these populations has greater diversity?



Nature 405, 212-219(11 May 2000)

The first has more species.
The second has its species more evenly distributed in terms of numbers.

Indicators of Biodiversity

- Traditional approaches consider two basic determinants of biodiversity:
  - ***Richness*** = number of species
  - ***Evenness*** = extent to which species are equally distributed
- Basic assumptions are:
  - All species are equal
  - All individuals are equal (we disregard differences in size, health, etc.)
  - Spatial distribution is irrelevant

Indicators of Biodiversity

- These may not be appropriate assumptions.
  - –Do we really want an ecosystem with as many leopards as zebras?
  - –Is a forest with 100 hemlock trees and 100 oak trees, well interspersed, equally diverse as a forest with 100 hemlocks in one half and 100 oaks in another half?

Indicators of Biodiversity

- These may not be appropriate assumptions.
  - Some species are highly “visible” or considered centrally important for conservation biology purposes (e.g., lions, elephants)
  - Some species are indicator species of the health of an ecosystem
    - Lichens indicate environmental stress; algal species may indicate organic pollution.
  - We may want to give the presence (or absence) of indicator species higher priority

Measuring Richness

- ***Richness S is usually interpreted as the number of different species in an ecosystem.***
- This has some major disadvantages:
  - –Doesn't pay attention to presence/absence of "important" or "indicator" species
  - –Is subject to sampling process to detect species
    - ➢Sampling process could be biased
    - ➢Could depend on length of time sampling is done – typical model
    - ➢Depends on area sampled
  - –Increases with presence of species we don't want to have (e.g., invasive species)

Measuring Richness

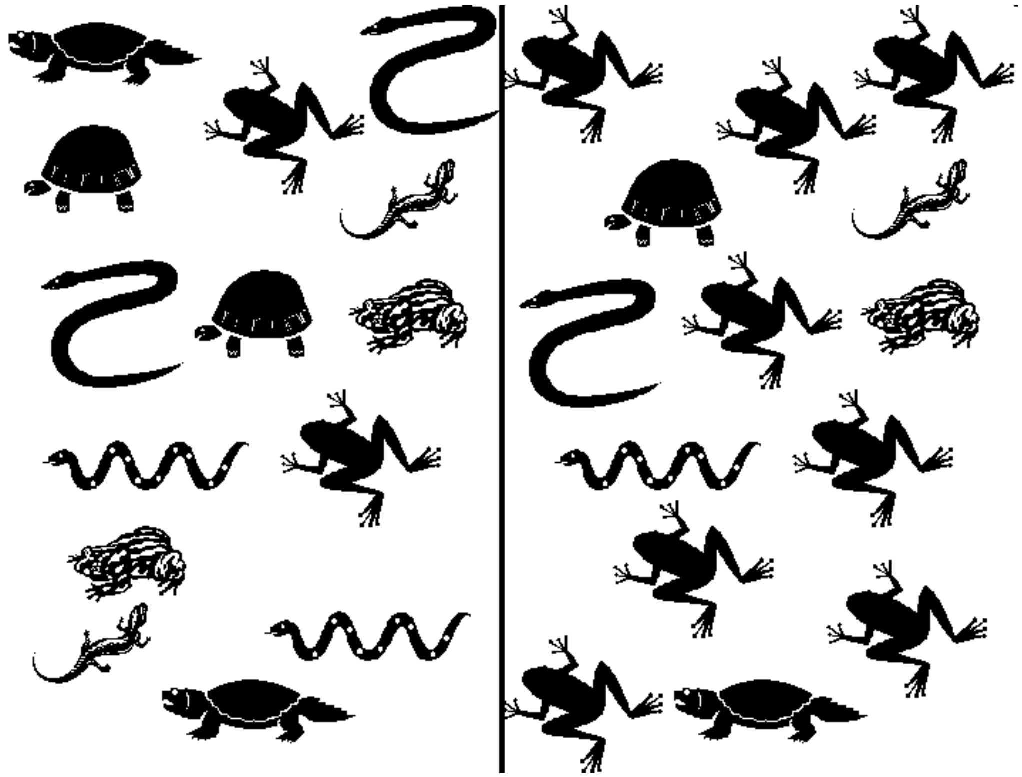
- Interesting mathematical approaches to the connection between time spent sampling and number of species detected.
- There is evidence that as time spent collecting increases, the number of species identified asymptotically approaches some limit.
- Models connecting richness to area sampled go back to 1921 (Arrhenius).

Measuring Evenness

- Based on ideas going back in the economic literature to the early 1900s.
- Work of Gini (1909, 1912) (measures of even income distribution)
- Work of Dalton (1920) (measures of inequality)
- Some measures of biodiversity or of evenness go back to work in communication theory, in particular work of Claude Shannon (1948) on entropy.
- Evenness also applied in other areas:
  - Linguistics
  - Distribution of academic publications

Measuring Evenness

- Which of these populations has greater biodiversity?



From Biodiversity, Oklahoma State Univ.

- Same number of species – so equally rich.
- Same population size.
- First has 2 of each species; second 1 of each except 8 frogs.
- First has as even a distribution as possible; second is highly uneven.

Some Sample Measures of Evenness

- S = number of species
- x_i = number of individuals of species i found = ***abundance*** of species i.
- In some cases, x_i is not number of individuals, but some measure of biomass.
- $x = (x_1,x_2,\ldots,x_S)$ = ***abundance vector***.
- ***Evenness measure*** = $f(x) = f(x_1,x_2,\ldots,x_S)$
- ***Take f(x) to be low if very even, high if very uneven***
- Often take $f(x)$ to be between 0 and 1
- In previous example, first population has abundance vector $x = (2,2,2,2,2,2,2)$; second has vector $x' = (1,1,1,1,1,1,8)$

Some Sample Measures of Evenness

- Let a_i = species i's proportion of the population.

$$a_i = x_i / \Sigma_j x_j$$

- Let $a = (a_1, a_2, \ldots, a_S)$
- In our example:
  - first population has $a = (1/7, 1/7, \ldots, 1/7)$
  - second population has $a' = (1/14, 1/14, \ldots, 1/14, 8/14)$.

- The literature has many proposed measures of evenness. We will give a few examples.

Simpson's Index

$$\lambda = \Sigma_i a_i^2$$

- This is the probability that any two individuals drawn at random from an infinite population will belong to the same species.
- In our example if $f(x) = \lambda(x)$:
  - For first population, $f(x) = (1/7)^2 + (1/7)^2 + \ldots + (1/7)^2 = 7/49 = 1/7 = .143$
  - For second population, $f(x') = (1/14)^2 + (1/14)^2 + \ldots + (1/14)^2 + (8/14)^2 = .357$
- Some biologists prefer high evenness to mean more even, and so use $1 - \lambda$ or $1/\lambda$ instead of λ.

Coefficient of Variation

$$V = \sigma/\mu$$

- μ = mean = $(1/S)\Sigma x_i$
- σ = standard deviation, where variance σ^2 is $(1/S)\Sigma(x_i-\mu)^2$.
- In our example if $f(x) = V(x)$:
  - For first population, all $x_i = \mu$, so $\sigma = 0$, and $f(x) = 0$.
  - For any population without perfectly even distribution, $f(x) > 0$.

Shannon-Wiener Diversity Index

$$H' = -\sum a_i \ln(a_i)$$

- In information theory, H' is called the ***Shannon entropy***.
- It quantifies (in expected value) the information contained in a message, in units such as bits.
- A fair coin has entropy of one bit.
- If coin is unfair and you are asked to bet, you will have less uncertainty.
- The Shannon index is maximized if each x_i is the same.
- We use $-H'$ so the index will be minimized if each x_i is the same.

Pielou Index

$$J = H'/H'_{max} = H'/\ln S$$

- H' is the Shannon-Wiener entropy.
- H'_{max} is the maximum value H' attains, i.e., $\ln S$, which occurs when all x_i are equal.
- We will use $-J$ so that the more even distribution of population gets the lower number.

Gini Index

$$G' = (2/S)[(S+1)/2 - \Sigma i a_i]$$

- Here, we assume that the x_i are ordered from high to low.
- In the literature, there are a number of variations of this.
- This measure was introduced in econometrics by Italian statistician Corrado Gini in 1909/1912.
- It is widely used to measure things like inequality of income or wealth distribution.
- G' = .25 for Denmark, .70 for Namibia. Higher G' means more uneven.

Gini Index

$$G' = (2/S)[(S+1)/2 - \Sigma i a_i]$$

- Note that if all the x_i are the same, then all a_i are the same, i.e., $1/S$.
- Then $\Sigma i a_i = \Sigma i(1/S) = (1/S)[(S/2)(S+1)] = (S+1)/2$
- Thus, $G' = 0$.
- In all other cases, $G' > 0$.

Which Index to Use?

•There are many other indices that have been proposed over the years.
•How does one choose?
•One idea is to write down some general principles (***axioms***) that a measure of evenness should satisfy and see which of the suggested indices satisfy them.
•One approach: Axioms originally due to Dalton (1920) in the economics literature and widely discussed in the literature of biodiversity.

Dalton's Axioms

The Principle of Permutation Invariance

Concentration or diversity or evenness is not a property of (names of) individual species but of a group of species considered as a whole.

If π is a permutation of $\{1,2,\ldots,S\}$, then

$$\mathbf{f(x_{\pi(1)},x_{\pi(2)},\ldots,x_{\pi(S)}) = f(x_1,x_2,\ldots,x_S).}$$

Dalton's Axioms

The Principle of Scale Invariance

A measure of concentration or diversity or evenness should not be influenced by the units used.

If c is a constant, then

$$\mathbf{f(cx_1,cx_2,\ldots,cx_S) = f(x_1,x_2,\ldots,x_S).}$$

Dalton's Axioms

The Transfer Principle

When the rich get richer and the poor get poorer, inequality rises.

If you increase the population of a more abundant species and decrease the population of a less abundant species, there is less evenness.

If $x_i < x_j$ and $0 < h \leq x_i$, then

$$\mathbf{f(x_1,x_2,\ldots,x_{i-1},x_i-h,x_{i+1},\ldots,x_{j-1},x_j+h,x_{j+1},\ldots,x_S) > f(x_1,x_2,\ldots,x_S).}$$

Measurement of Biodiversity

- The five indices we have defined satisfy all three Dalton axioms.
- So, how do we choose among them?
- We try to add more conditions that a measure should satisfy.

Measurement of Biodiversity

If the Rich Get Richer Principle

If x_i is the maximum of $\{x_1, x_2, \ldots, x_S\}$, then for every $h > 0$,

$$\mathbf{f(x_1, x_2, \ldots, x_{i-1}, x_i + h, x_{i+1}, \ldots, x_S) > f(x_1, x_2, \ldots, x_S)}$$

Measurement of Biodiversity

The Principle of Nominal Increase

If not all x_i are equal, then for every $h>0$,

$$f(x_1+h, x_2+h, \ldots, x_S+h) < f(x_1, x_2, \ldots, x_S)$$

If everyone receives the same nominal increase in salary, there is less inequality (f decreases).

Measurement of Biodiversity

Theorem (Egghe and Rousseau 1991). The If the Rich get Richer Principle and the Principle of Nominal Increase follow from the earlier Dalton Axioms of Permutation Invariance, Scale Invariance and the Transfer Principle.

Thus, all five of the indices we have described satisfy these two axioms also.

Measurement of Biodiversity

The Replication Principle

If a population is replicated and a new population consists of the old plus the new, then the evenness should not change.

•Example: (3,7) and (3,7,3,7) should have the same evenness.

•Simpson index, Shannon-Wiener, and Pielou violate replication. However, Gini and Coefficient of Variation satisfy it.

Another Approach: The Lorenz Partial Order

- The axioms show that certain abundance vectors are clearly less evenly distributed than others.
- We introduce an order relation $\prec$
- If x and x' are two abundance vectors, $x \prec x'$ means that the evenness of x is less than the evenness of x'. (This means x is ***more even*** than x'.)
- We want this to be a ***partial order***
- Goal: the evenness measure should reflect this partial order:

$$x \prec y \Longrightarrow f(x) < f(y)$$

Another Approach: The Lorenz Partial Order

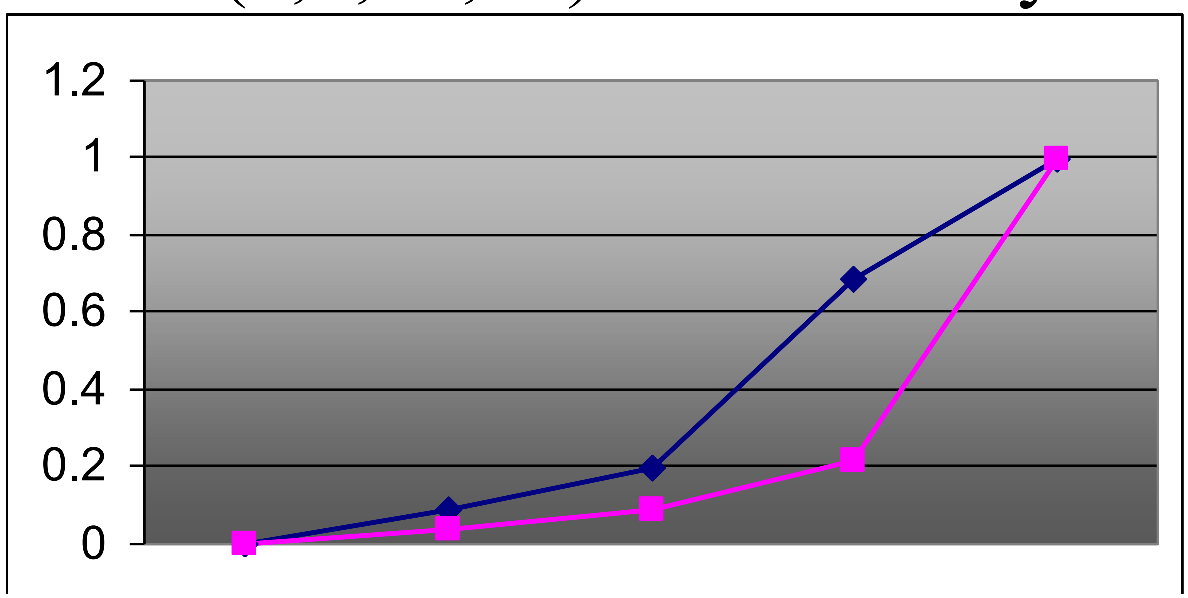
- The ***Lorenz curve*** of an abundance vector x is obtained as follows:
- Assume that $x_1 \leq x_2 \leq \ldots \leq x_S$.
- Let $b_j = a_1 + a_2 + \ldots + a_j$
- Then b_j is the cumulative proportion of the population due to the first j species.
- Locate S+1 points in (x,y)-space by using the points

$$(0,0), (1/S,b_1), (2/S,b_2), \ldots, (S/S,b_S)$$

- Join these points by straight lines

The Lorenz Partial Order

- Sometimes the curve for one abundance vector x is strictly above the curve for another vector y at all points, i.e., $x_i > y_i$ for all i.
- In such a case, this corresponds to the evenness of x being less than that of y, i.e., we take $x \prec y$. (So x is more even than y.)
- Here are curves for x = (5,6,20,25) in blue and y = (2,3,7,44) in pink.



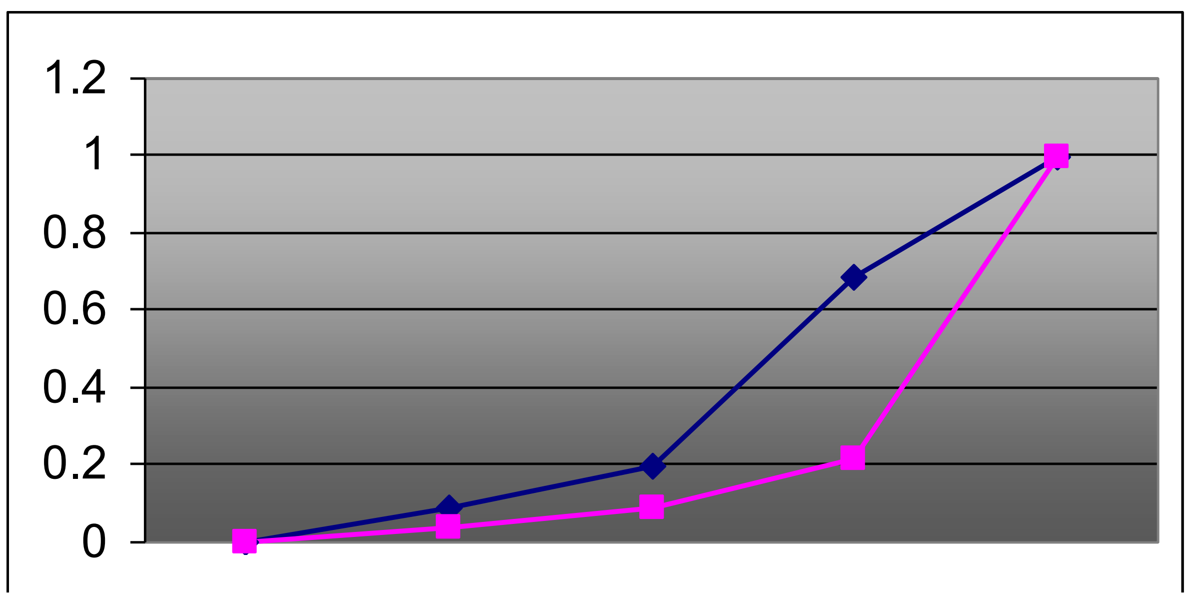
The Lorenz Partial Order

•The Gini Index captures this partial order:

$$x \prec y \Longrightarrow f(x) < f(y)$$

•Many research challenges remain, e.g., can we combine richness and evenness?

•E.g., can we design ecosystems of relatively stable biodiversity?



Concluding Comments

- Only by putting the measurement of biodiversity on a firm mathematical foundation can we be confident that we are capturing the true diversity in nature.

Cicada images courtesy of Nina Fefferman

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MPE: Management of Natural Resources
How can Math Sciences help? How can we each help?