The Privacy of Secured Computations

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"Relax – it can only see metadata."



Big Data

Every <length of time>
your <household object>
generates <metric scale modifier>bytes of data
about you

- Everyone handles sensitive data
- Everyone delegates sensitive computations



Secured computations

- Modern crypto offers powerful tools
 - Zero-knowledge to program obfuscation
- Broadly: specify outputs to reveal
 - … and outputs to keep secret> Reveal only what is necessary
- Bright lines
 - E.g., psychiatrist and patient
- Which computations should we secure?
 - Consider average salary in department before and after professor X resigns
 - Today: settings where we must release some data at the expense of others





Which computations should we secure?

This is a social decision
 ➤ True, but...



 Technical community can offer tools to reason about security of secured computations



- This talk: privacy in statistical databases
- Where else can technical insights be valuable?



Large collections of personal information

- census data
- national security data
- medical/public health data
- social networks
- recommendation systems
- trace data: search records, etc

Two conflicting goals

Utility: Users can extract "aggregate" statistics
 "Privacy": Individual information stays hidden

• How can we define these precisely?

Variations on model studied in

- Statistics ("statistical disclosure control")
- Data mining / database ("privacy-preserving data mining" *)
- Recently: Rigorous foundations & analysis

Why is this challenging?

> A partial taxonomy of attacks

Differential privacy

"Aggregate" as insensitive to individual changes

Connections to other areas

External Information



Users have external information sources
 Can't assume we know the sources

Anonymous data (often) isn't.

A partial taxonomy of attacks

Reidentification attacks

Based on external sources or other releases

- Reconstruction attacks
 - "Too many, too accurate" statistics allow data reconstruction
- Membership tests
 - Determine if specific person in data set (when you already know much about them)









- Correlation attacks
 - Learn about me by learning about population

Reidentification attack example

[Narayanan, Shmatikov 2008]





Alice Bob Charlie Danielle Erica Frank

Anonymized NetFlix data

Public, incomplete



On average, four movies uniquely identify user

Identified NetFlix Data

Other reidentification attacks

- ... based on external sources, e.g.
 - Social networks
 - Computer networks
 - Microtargeted advertising
 - Recommendation Systems
 - Genetic data [Yaniv's talk]
- ... based on composition attacks
 Combining independent anonymized releases

e.g.



[Citations omitted]

Is the problem granularity?

- Examples so far: releasing individual information
 What if we release only "aggregate" information?
- Defining "aggregate" is delicate
 - E.g. support vector machine output reveals individual data points



Statistics may together encode data

Reconstruction attacks:
 Too many, "too accurate" stats
 reconstruct the data

Robust even to fairly significant noise

Reconstruction Attack Example [Dinur Nissim '03]

Data set: d "public" attributes, I "sensitive"



- Suppose release reveals correlations between attributes
 - > Assume one can learn $\langle a_i, y \rangle + error$
 - For $rac{i}{i}$ of $rac{i}{i}$ of a_i uniformly random and d > 4n, then one reconstruct n - o(n) entries of y

Too many, "too accurate" stats ⇒ reconstruct data
 Cannot release everything everyone would want to know

Reconstruction attacks as linear encoding [DMT'07,...]

• Data set: d "public" attributes per person, I "sensitive"



• Idea: view statistics as noisy linear encoding My + e



Reconstruction depends on geometry of matrix M

> Mathematics related to "compressed sensing"

Membership Test Attacks

 [Homer et al. (2008)]
 Exact high-dimensional summaries allow an attacker
 with knowledge of population
 to test membership in a data set



Membership is sensitive

> Not specific to genetic data (no-fly list, census data...)

> Learn much more if statistics are provided by subpopulation

• Recently:

Strengthened membership tests [Dwork, S., Steinke, Ullman, Vadhan '15]

Tests based on learned face recognition parameters [Frederiksson et al '15]

Membership tests from marginals

- X: set of n binary vectors from distrib P over $\{0,1\}^d$
- $q(X) = \overline{X} \in [0,1]^d$: proportion of 1 for each attribute
- $z \in \{0,1\}^d$: Alice's data
- Eve wants to know if Alice is in X.
 Eve knows

$$\succ q(X) = \overline{X}$$

 \succ z: either in X or from P

 \succ Y: n fresh samples from P

• [Sankararam et al, '09] Eve reliably guesses if $z \in X$ when d > cn







Strengthened membership tests [DSSUV'15]

- X: set of n binary vectors from distrib P over $\{0,1\}^d$
- $q(X) = \overline{X} \pm \alpha$: approximate proportions
- $z \in \{0,1\}^d$: Alice's data
- Eve wants to know if Alice is in X.
 Eve knows

$$\succ q(X) = \overline{X} \pm \alpha$$

$$\succ z$$
: either in X or from P

 \succ Y: **m** fresh samples from P

[DSSUV'15] Eve reliably guesses if $z \in X$ when $d > c' \left(n + \alpha^2 n^2 + \frac{n^2}{m} \right)$

X =

$$q(X) \approx$$
1/2 3/4 1/2 1/2 3/4 1/2 1/4 1/2

$$Z = 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0$$

Robustness to perturbation



Two publication mechanisms

Rounded to nearest multiple of 0.1 (red / green)

Exact statistics (yellow / blue)

Conclusion: IP test is robust. Calibrating LR test seems difficult

"Correlation" attacks

Suppose you know that I smoke and...

- Public health study tells you that I am at risk for cancer
- You decide not to hire me



- Learn about me by learning about underlying population
 - It does not matter which data were used in study
 - \succ Any representative data for population will do

Widely studied

- De Finetti [Kifer '09]
- Model inversion [Frederickson et al '15] *
- Many others

Correlation attacks fundamentally different from others

- Do not rely on (or imply) individual data
- \succ Provably impossible to prevent **

* Model inversion used two few different ways in [Frederickson et al.] ** Deta

A partial taxonomy of attacks

Reidentification attacks

Based on external sources or other releases

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- Correlation attacks
 - Learn about me by learning about population

• Why is this challenging?

➢ A partial taxonomy of attacks



• Intuition:

Changes to my data not noticeable by users

> Output is "independent" of my data



- Data set x = (x₁, ..., x_n) ∈ Dⁿ
 Domain D can be numbers, categories, tax forms
 Think of x on fixed (not non dom)
 - Think of x as fixed (not random)
- A = **randomized** procedure
 - > A(x) is a random variable
 - > Randomness might come from adding noise, resampling, etc.



• A thought experiment

- > Change one person's data (or remove them)
- > Will the distribution on outputs change much?



x' is a neighbor of x if they differ in one data point

Definition: A is ε -differentially private if,

Neighboring databases induce **close** distributions on outputs

for all neighbors x, x',

for all subsets S of outputs

$$\Pr(\mathsf{A}(\mathsf{x}) \in \mathsf{S}) \le e^{\epsilon} \cdot \Pr(\mathsf{A}(\mathsf{x}') \in \mathsf{S})$$



• This is a condition on the **algorithm** A

Saying a particular output is private makes no sense

- Choice of distance measure matters
- What is <mark>ɛ</mark>?

Measure of information leakage

$$\blacktriangleright$$
 Not too small (think $rac{1}{10}$, not $rac{1}{2^{50}}$)

Definition: A is ε -differentially private if, for all neighbors x, x', for all subsets S of outputs $Pr(A(x) \in S) \leq e^{\epsilon} \cdot Pr(A(x') \in S)$

Neighboring databases



- Say we want to release a summary $f(x) \in \mathbb{R}^p$ \triangleright e.g., proportion of diabetics: $x \in \{0,1\}$ and $f(x) = \frac{1}{n} \sum_i x_i$
- Simple approach: add noise to f(x)
 ➢ How much noise is needed?
- Intuition: f(x) can be released accurately when f is insensitive to individual entries x_1, \dots, x_n







- Example: proportion of diabetics
 - > GS_{proportion} = $\frac{1}{n}$ > Release A(x) = proportion ± $\frac{1}{\epsilon n}$
- Is this a lot?
 - ➢ If x is a random sample from a large underlying population, then sampling noise ≈ 1/√n
 ➢ A(x) "as good as" real proportion

0.5

-0.5

Useful Properties

Composition:

If A_1 and A_2 are ε -differentially private, then joint output (A_1 , A_2) is 2ε -differentially private.

- Post processing: A is ε-differentially private, then so is g(A) for any function g
- Meaningful in the presence of arbitrary external information

Definition: A is ε -differentially private if, for all neighbors x, x', for all subsets S of outputs $Pr(A(x) \in S) \leq e^{\epsilon} \cdot Pr(A(x') \in S)$

Interpreting Differential Privacy

A naïve hope:

Your beliefs about me are the same after you see the output as they were before

- Impossible because of correlation attacks
- **Theorem [DN'06]**: Learning things about individuals is unavoidable in the presence of external information

Differential privacy implies: No matter what you know ahead of time,

You learn (almost) the same things about me whether or not my data are used

Features or bugs?

- May not protect sensitive global information, e.g.
 Clinical data: Smoking and cancer
 Financial transactions: firm-level trading strategies
 - > Social data: what if my presence affects everyone else?
- Leakage accumulates with composition
 - $\succ \epsilon$ adds up with many releases
 - Inevitable in some form [reconstruction attacks]
 - \succ How do we set ϵ ?

Variations on the approach

- Predecessors [DDN'03,EGS'03,DN'04,BDMN'05]
- (ε,δ)- differential privacy
 > Require Pr(A(x) ∈ S) ≤ e^ε · Pr(A(x) ∈ S) + δ
 > Similar semantics to (ε,0)- diffe.p. when δ ≪ 1/n
- Computational variants [MPRV09, MMPRTV'10, GKY'11]
- Distributional variants [RHMS'09,BBGLT'11,BD'12,BGKS'13]
 - > Assume something about adversary's prior distribution
 - Deterministic releases
 - Composition becomes delicate
- Generalizations
 - [BLR'08, GLP'II] simulation-based definitions
 - [KM'12, BGKS'13] General language for specifying privacy concerns. Downside: tricky to instantiate

What can we compute privately?



 "Privacy" = change in one input leads to small change in output distribution

What computational tasks can we achieve privately?

• Lots of recent work, interesting questions

Across different fields: statistics, data mining, machine learning, cryptography, algorithmic game theory, networking, info. theory

A Broad, Active Field of Science

- Basic Tools and Techniques
- Implemented systems
 - RAPPOR (Google)
 - PInQ (Microsoft)
 - ➢ Fuzz (U. Penn)
 - Privacy Tools (Harvard)
- Theoretical Foundations
 - Feasibility results: Learning, optimization, synthetic data, statistics
 - Connections to game theory, robustness, false discovery
- Domain-specific algorithms

> Networking, clinical data, social networks, ...



Basic Technique 1: Noise Addition



Example: Noise Addition [Dwork, McSherry, Nissim, S.



Example: Histograms

Say x₁,x₂,...,x_n in domain D
Partition D into d disjoint bins
f(x) = (n₁, n₂,..., n_d) where n_j = #{i : x_i in *j*-th bin}
GS_f = I

 \succ Sufficient to add noise Lap $(1/\epsilon)$ to each count

Examples

- ➢ Histogram on the line
- Populations of 50 states
- Marginal tables
 - bins = possible combinations of attributes



ABO and Rh Blood Type Frequencies in the United States

ABO Type	Rh Type positive	How Many Have It	
0		38%	4504
0	negative	7%	45%
Α	positive	34%	40%
A	negative	6%	
В	positive	9%	11%
В	negative	2%	
AB	positive	3%	4%
AB	negative	1%	

Using global sensitivity

$$\mathsf{GS}_{f} = \max_{\text{neighbors } x, x'} \|f(x) - f(x')\|_{1}$$

- Many natural functions have low sensitivity
 - e.g., histogram, mean, covariance matrix, distance to a function, estimators with bounded "sensitivity curve", strongly convex optimization problems
- Laplace mechanism can be a programming interface [BDMN '05]
 - Implemented in several systems [McSherry '09, Roy et al. '10, Haeberlen et al. '11, Moharan et al. '12]

Variants in other metrics

- Consider $f : \mathcal{D}^n \to \mathbb{R}^d$
- Global Sensitivity: $GS_f = \max_{\text{neighbors } x, x'} \|f(x) f(x')\|_{\frac{4}{2}}$

Theorem: If $A(x) = f(x) + L_{2p} \left(\frac{cs}{c}\right)^{d}$, then A is *d* differentially private.

• Example
$$N\left(0, \left(\frac{GS_f \cdot 3 \cdot \sqrt{\ln(1/\delta)}}{\epsilon}\right)^2\right)$$
 (ϵ, δ)
> $f(x) = \text{vector of counts.}$
> $Add noise \overline{d}$ per entry instead of
 $\frac{\sqrt{d\ln(1/\delta)}}{\epsilon}$ $\frac{d}{\epsilon}$

Global versus local [NRS07]



- Global sensitivity is worst case over inputs
- Local sensitivity:

$$\mathsf{LS}_f(x) = \max_{\mathbf{x'} \text{ neighbor of } x} \|f(x) - f(\mathbf{x'})\|_1$$

- Reminder:
- [NRS'07,DL(x) Techniques with error \approx local sensitivity

Basic Technique 2: Exponential Sampling



Exponential Sampling [McSherry, Talwar '07]

- Sometimes noise addition makes no sense
 - ➤ mode of a discrete distribution
 - minimum cut in a graph
 - classification rule
- [MT07] Motivation: auction design
- Subsequently applied very broadly

Example: Popular Sites

- Data: x_i = {websites visited by student i today}
- Range: Y = {website names}
- "Score" of y: $q(y; x) = |\{i : y \subseteq x_i\}|$
- Goal: output the most frequently visited site

Mechanism: Given x,

- Output website y_0 with probability $r_x(y) \propto \exp(\epsilon q(y; x))$
- Utility: Popular sites exponentially more likely than rare ones
- Privacy: One person changes websites' scores by ≤I

 $q(y; \mathsf{x})$

Analysis

Mechanism: Given x,

- Output website y_0 with probability $r_x(y) \propto \exp(\epsilon q(y; x))$
- **Claim:** Mechanism is 2ε-differentially private
- Proof: $\frac{r_{\mathsf{x}}(y)}{r_{\mathsf{x}'}(y)} = \frac{e^{\epsilon q(y;\mathsf{x})}}{e^{\epsilon q(y;\mathsf{x}')}} \cdot \frac{\sum_{z \in Y} e^{\epsilon q(z;\mathsf{x}')}}{\sum_{z \in Y} e^{\epsilon q(z;\mathsf{x})}} \le e^{2\epsilon}$
- Claim: If most popular website has score T, then $\mathbb{E}[q(y_0;x)] \geq T (\log |Y|)/\epsilon$
- Proof: Output y is bad if q(y;x) < T k

$$\Pr(\text{bad outputs}) \le \frac{\Pr(\text{bad outputs})}{\Pr(\text{best output})} \le \frac{|Y|e^{\epsilon(T-k)}}{e^{\epsilon T}} \le e^{\log|Y| - \epsilon k}$$

Set expectation bound via formula $E(Z) = \sum_{k>0} \Pr(Z \ge k)$

Exponential Sampling

Ingredients:

- Set of outputs Y with prior distribution p(y)
- Score function q(y;x) such that for all outputs y, neighbors x,x': |q(y;x) - q(y;x')| ≤ |
 Mechanism: Given x,
 Output y₀ from Y with probability r_x(y) ∝ p(y)e^{εq(y;x)}
- Basis for first synthetic data results [Blum, Ligett, Roth '08]
 Preserve k linear statistics about data set with domain D

$$\frac{(\log^{1/2} k)(\log^{1/4} |D|)}{n^{1/2}}$$

Using Exponential Sampling

Mechanism above very general

Every differentially private mechanism is an instance!

- > Still a useful design perspective
- Perspective used explicitly for
 - Learning discrete classifiers [KLNRS'08]
 - > Synthetic data generation [BLR'08,...,HLM'10]
 - Convex Optimization [CM'08,CMS'10]
 - Frequent Pattern Mining [BLST'10]
 - Genome-wide association studies [FUS'11]
 - > High-dimensional sparse regression [KST'12]

Digital Good Auction [McSherry, Talwar '07]

• I seller with a digital good

• n potential buyers



- \succ Each has a secret value v_i in [0,1] for song
- > Setting price p will get revenue $rev(p) = p|\{i: vi \ge p\}|$
- \succ How can seller set p to get revenue \approx OPT = max rev(p)?
- Straightforward bidding mechanism
 - Each player reports vi'
 - > Lying can drastically change best price
- Instead, sample p* from density r(p) ∝ exp(ε.rev(p))
 > Expected revenue ≥ OPT O(ln(εn) / ε)

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Implications for other areas

- Game theory & economics
 - Differentially private mechanisms are automatically "approximately truthful"
 - > Participating in a DP mechanism doesn't hurt me
- Statistical analysis: Differential privacy is a strong type of stability or robustness
 - Regularization techniques from optimization help design DP algorithms
 - > Control false discovery in adaptive data analysis

Ongoing Work

- Practical implementations
- Efficient algorithms
- Relaxed definitions
 - Exploit adversarial uncertainty
- Differently-structured data

E.g., social network data: which data is "mine"?

Conclusions

- Define privacy in terms of my effect on output
 Meaningful despite arbitrary external information
 - > I should participate if I get benefit
- Rigorous framework for private data analysis
 Rich algorithmic literature (theoretical and applied)
 There is no competing theory
- What computations can we secure?
 - Differential privacy provided a surprising formalization for a previously ad hoc area
 - What other areas need formalization?
 - How should we think about correlation attacks?

Further resources

- Tutorial from CRYPTO 2012
 - http://www.cse.psu.edu/~asmith/talks/2012-08-21-cryptotutorial.pdf
- Courses:
 - <u>http://www.cis.upenn.edu/~aaroth/courses/privacyFII.html</u>
 - <u>http://www.cse.psu.edu/~asmith/privacy598</u>
- DIMACS Workshop on Data Privacy (October 2012)
 > Videos of tutorials
 - <u>http://dimacs.rutgers.edu/Workshops/DifferentialPrivacy/</u>
- Simons Institute Big Data & DP Workshop (Dec 2013)
 Talk videos online