Delegation with (nearly) optimal time/space overhead

Justin Holmgren
MIT

Ron Rothblum
MIT

Verifiable Computation

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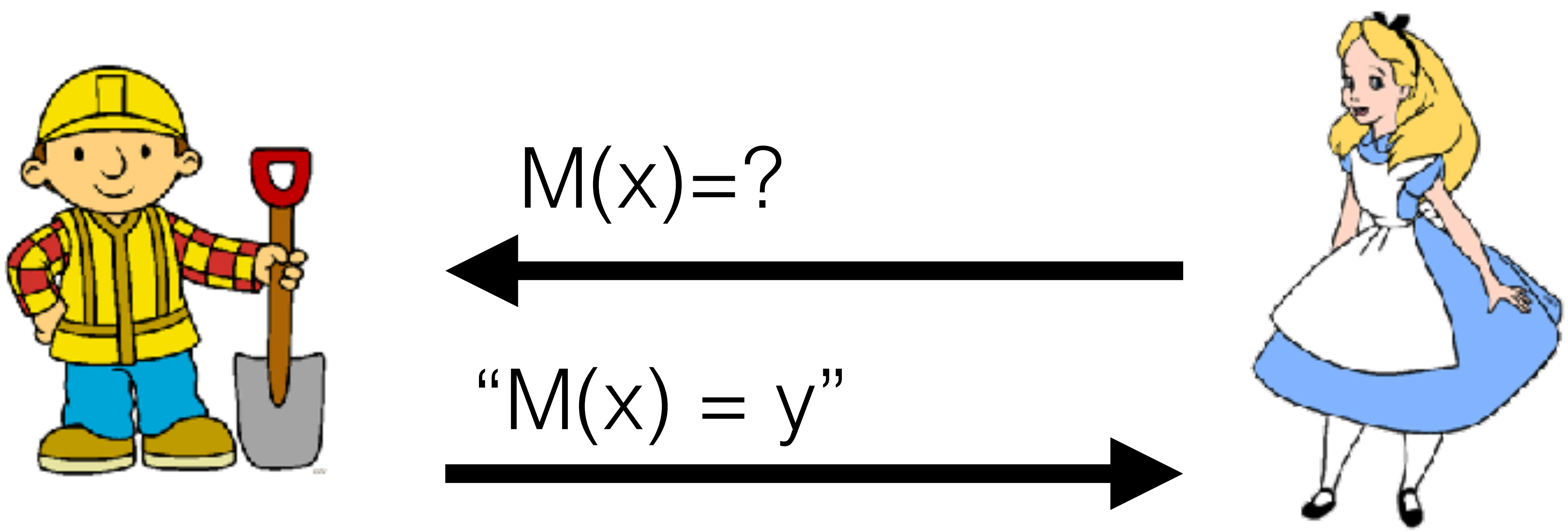

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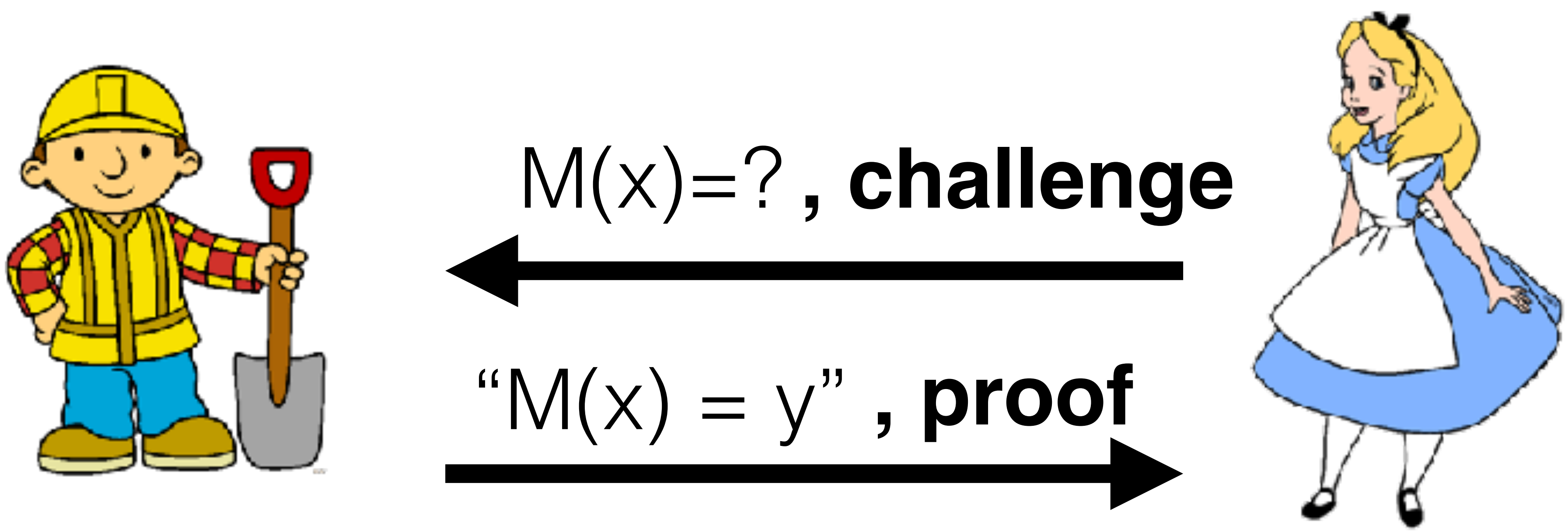

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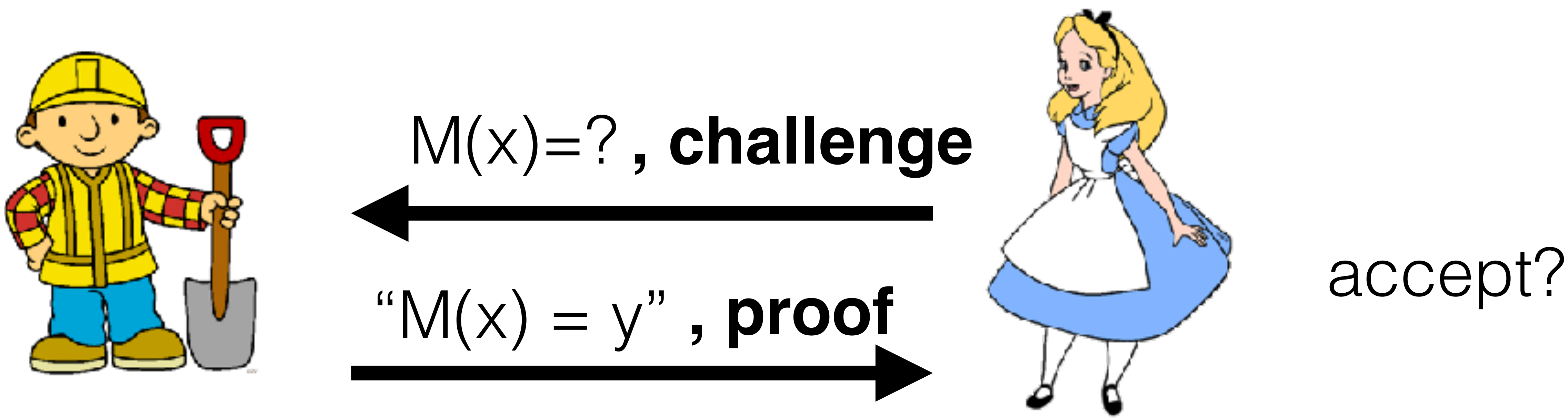

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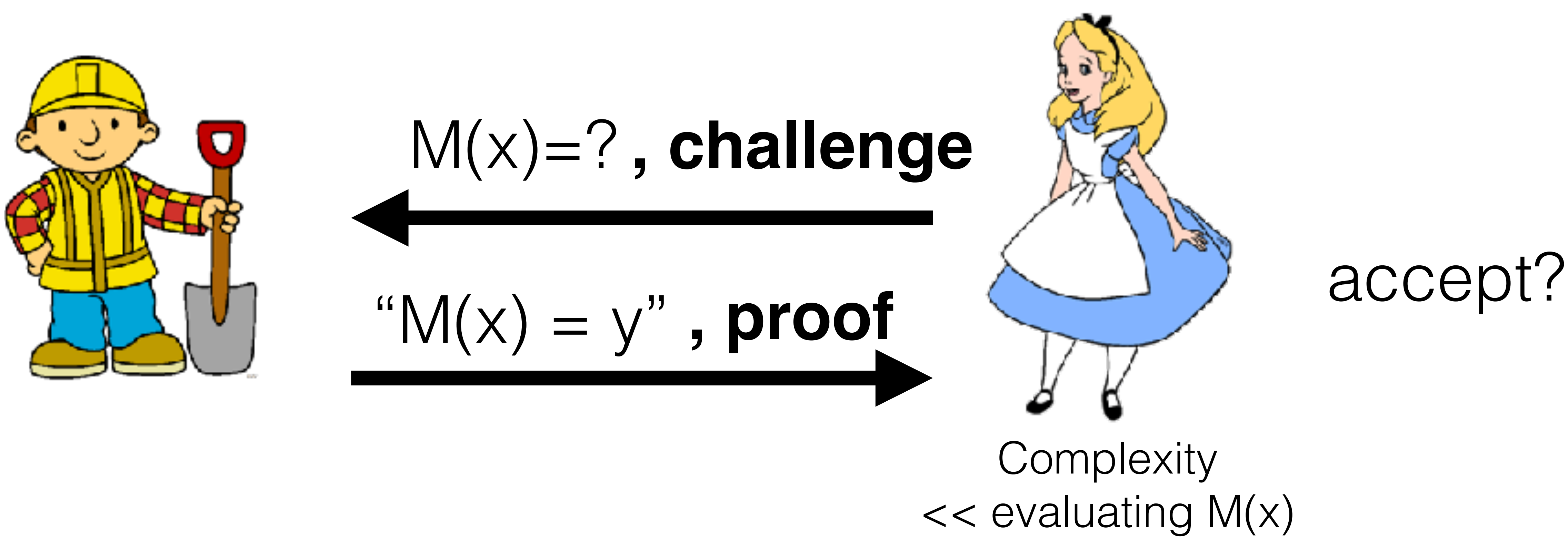


Verifiable Computation

Complexity
~evaluating M(x)

M(x)=? **, challenge**

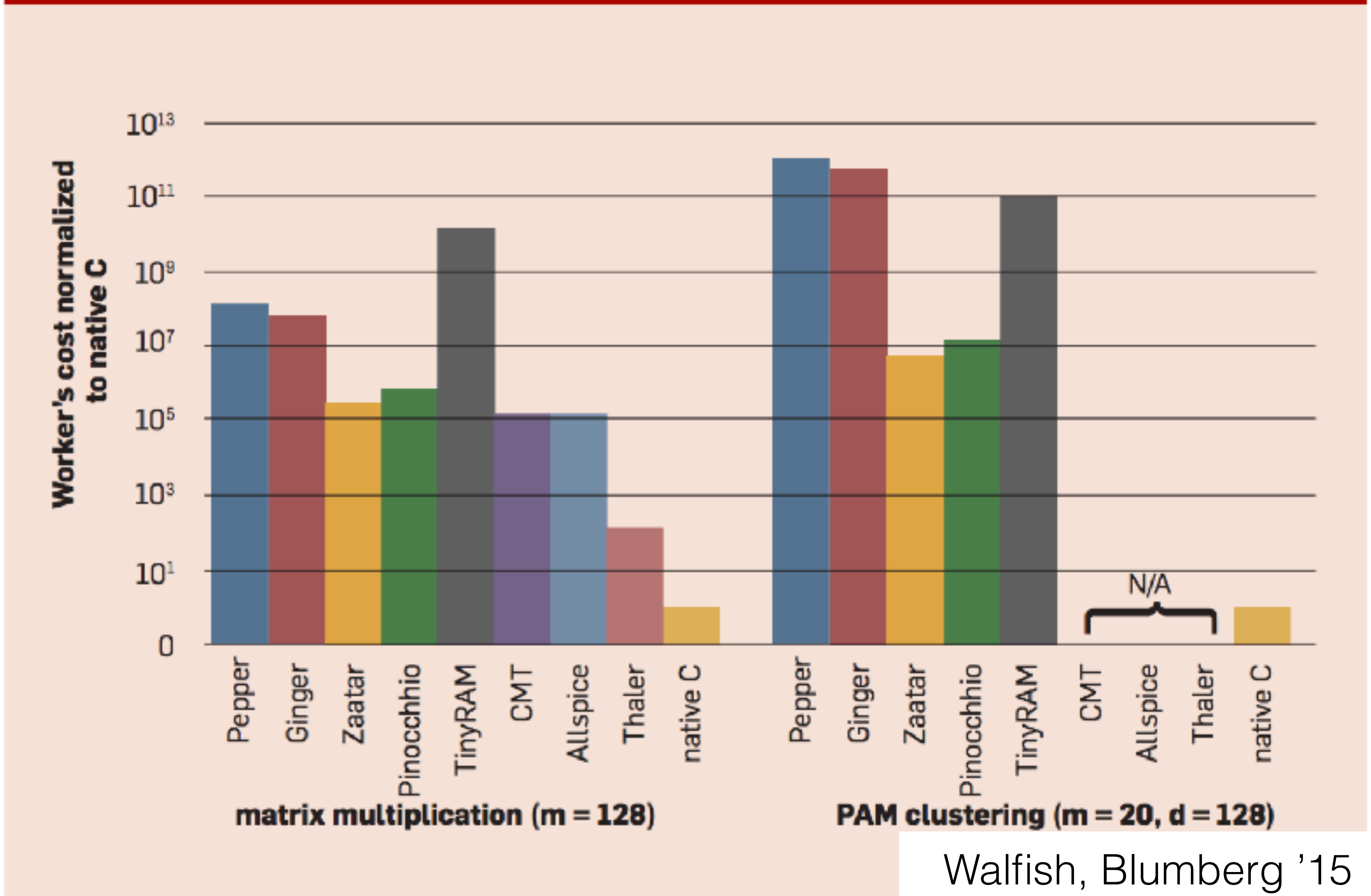
"M(x) = y" **, proof**

Complexity
<< evaluating M(x)

accept?

Verifiable Computation In Practice

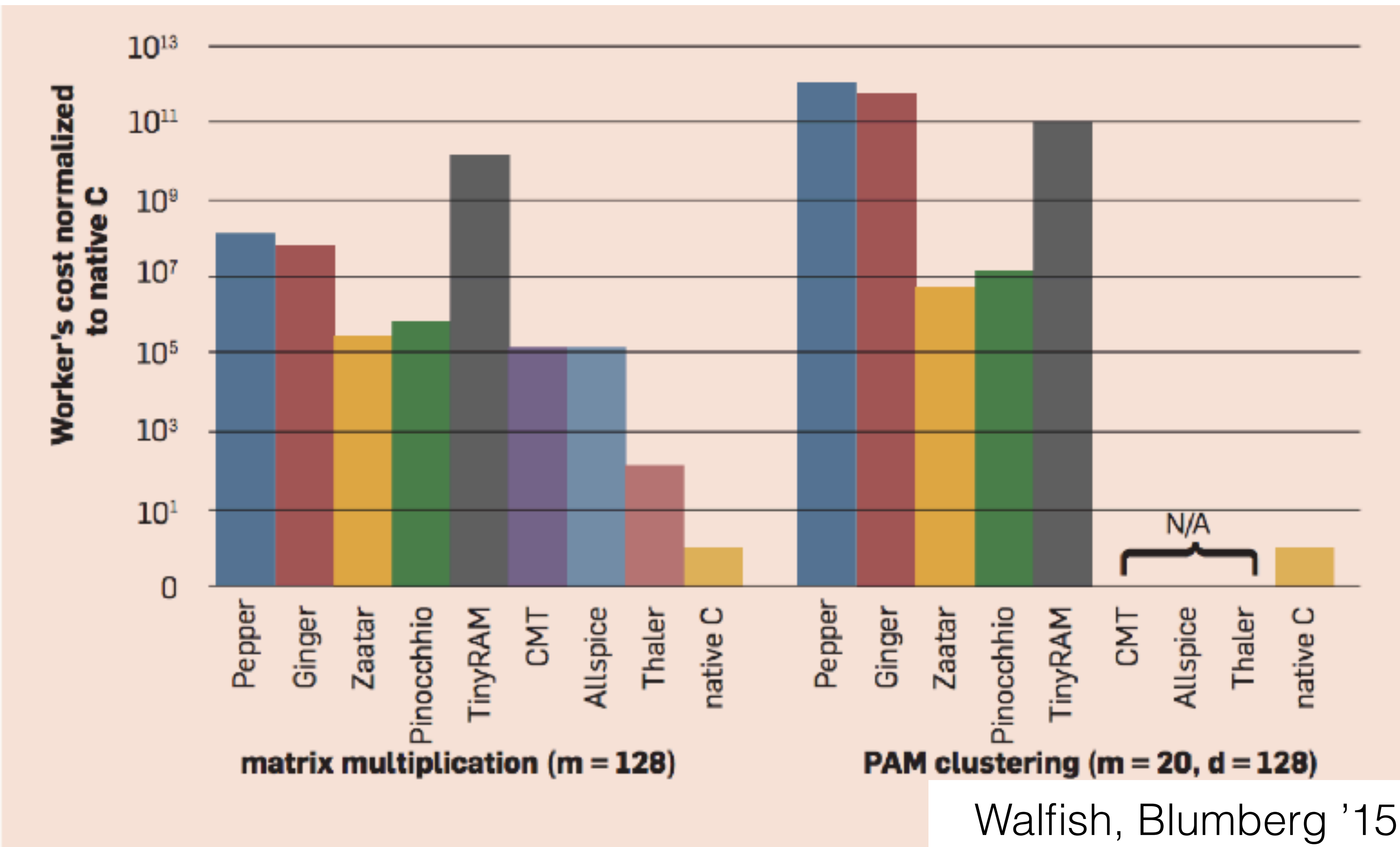
Figure 5. Prover overhead normalized to native execution cost for two computations. Prover overheads are generally enormous.



Walfish, Blumberg '15

Verifiable Computation In Practice

"An additional bottleneck is memory: the prover must materialize a transcript of a computation's execution."



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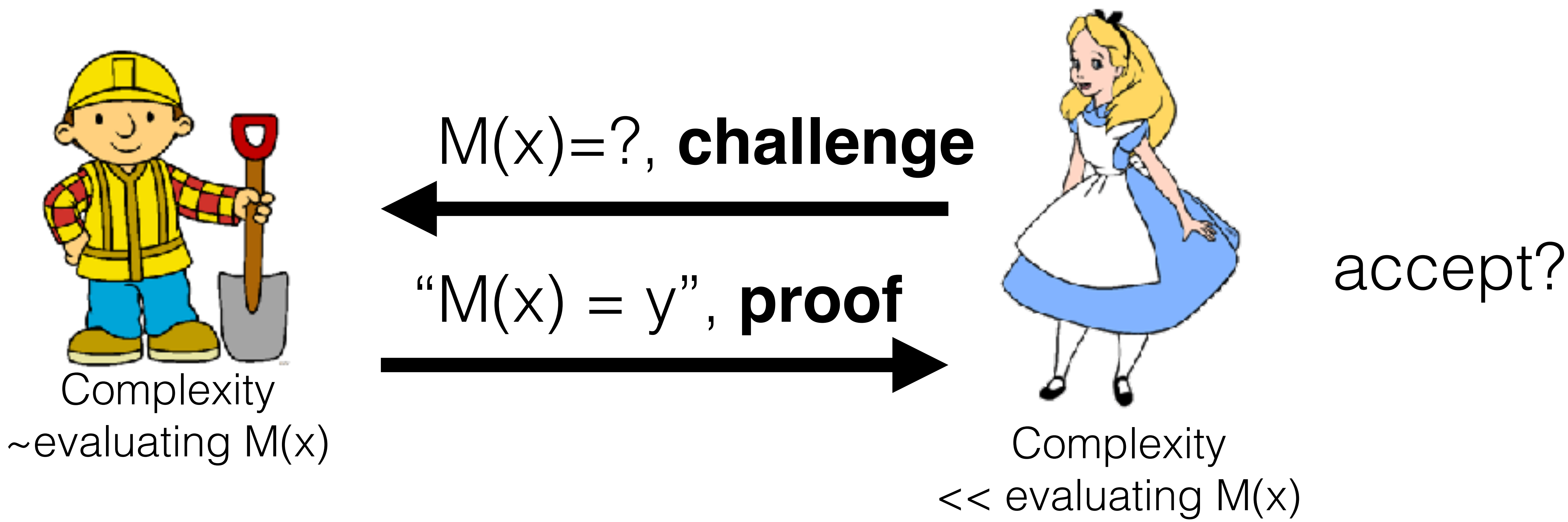

Our focus:

- Prover efficiency
- Computational assumptions

Prior Work

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	Model	Assumptions	Prover Time	Prover Space
No-Signaling PCP [KRR14, KP15, B**H**K16]	RAM	PIR	poly(T)	poly(T)

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Succinct Garbling [GHRW14, KLW15, C**H**15, CCCLLZ15]	RAM	Obfuscation	$T \cdot \text{poly}(\kappa)$	$S \cdot \text{poly}(\kappa)$

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Probabilistically Checkable Proofs

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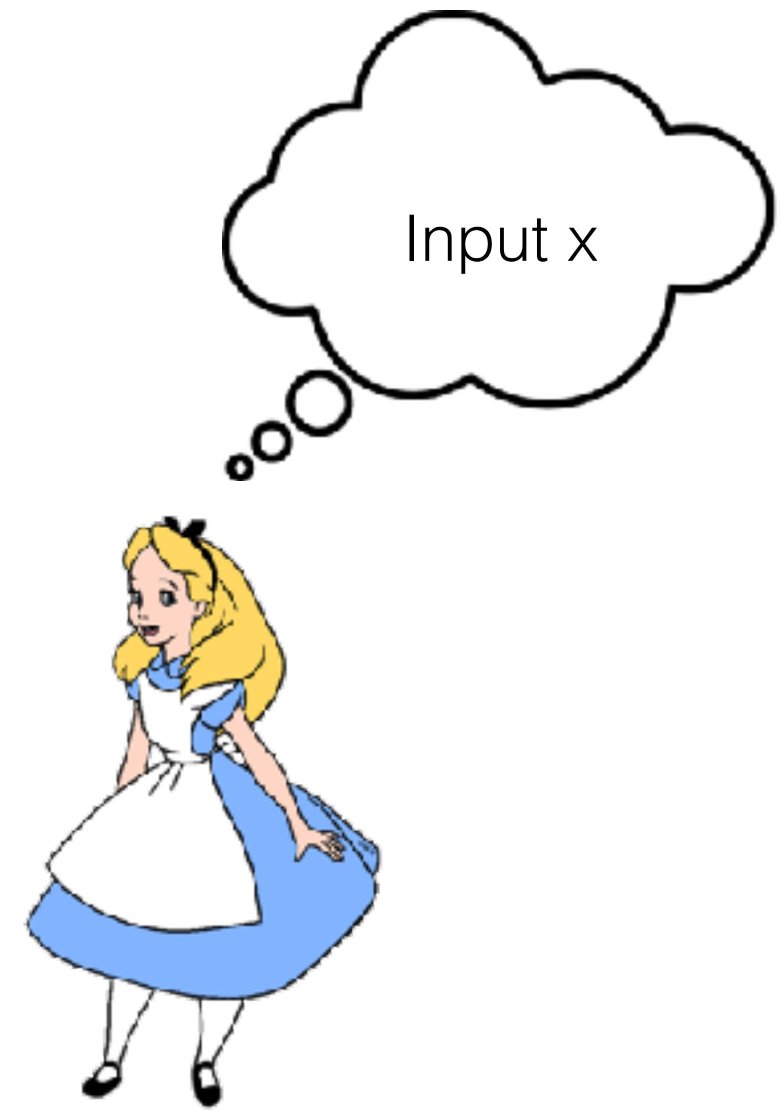
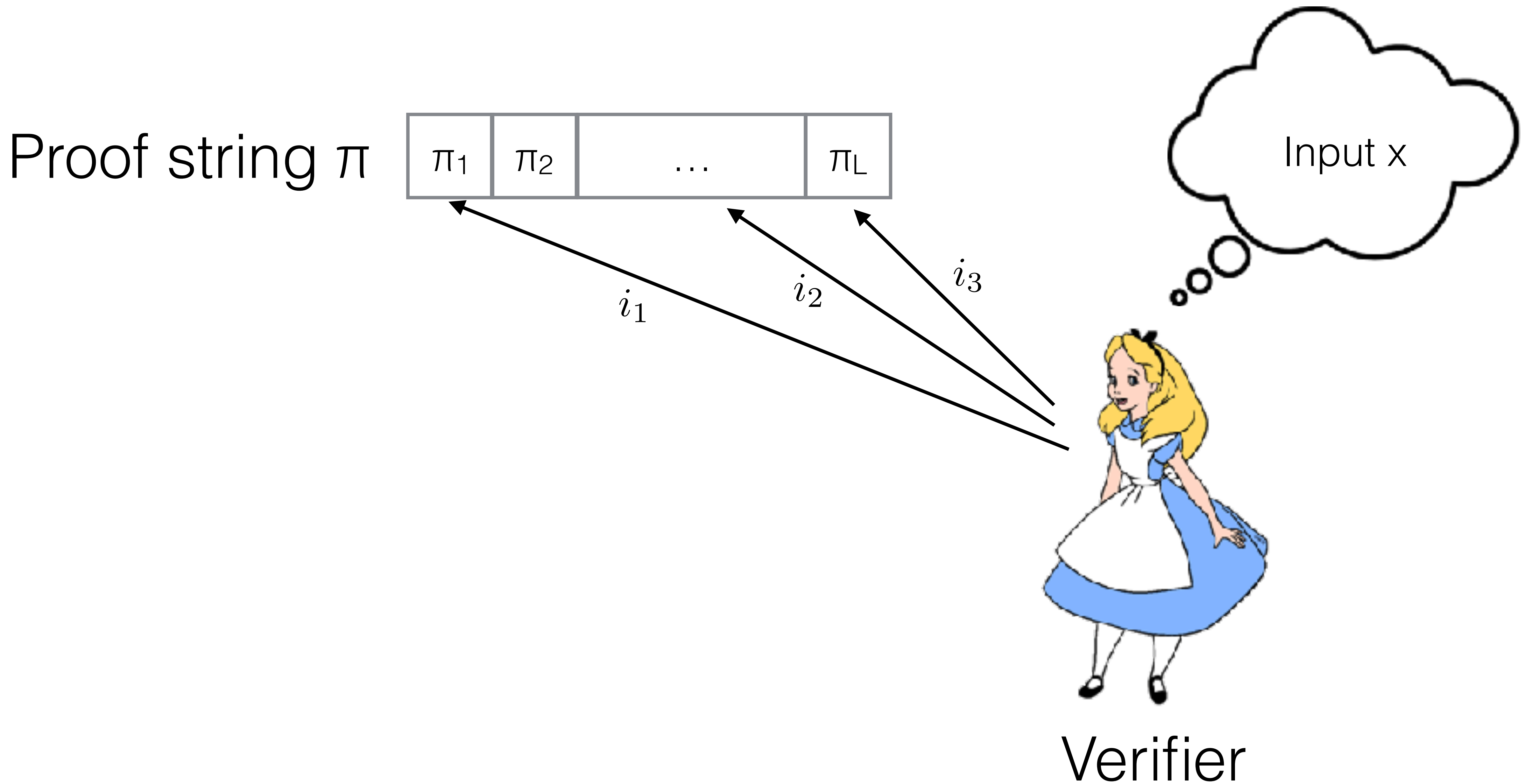
Proof string π

π_1	π_2	…	π_L



Verifier

Probabilistically Checkable Proofs



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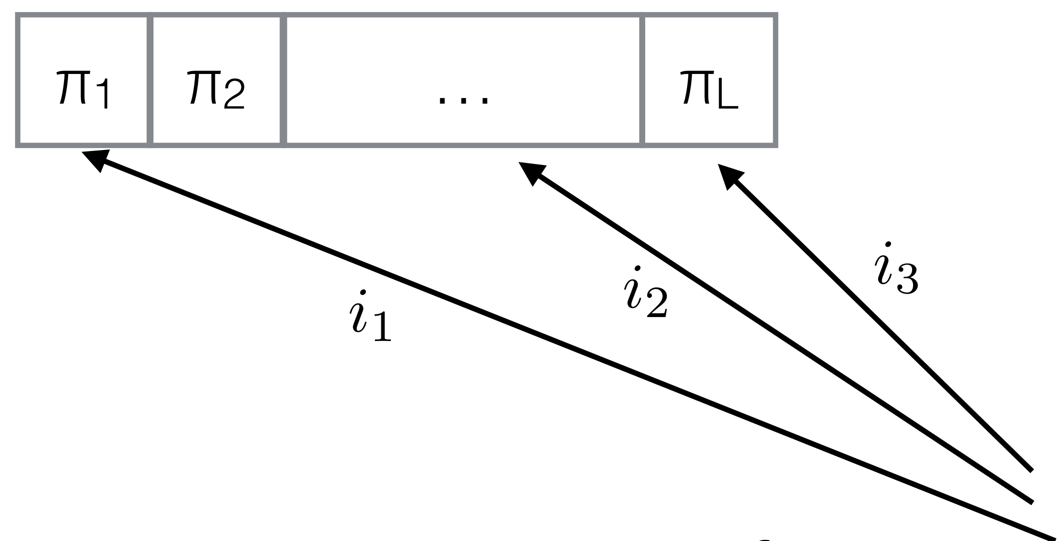

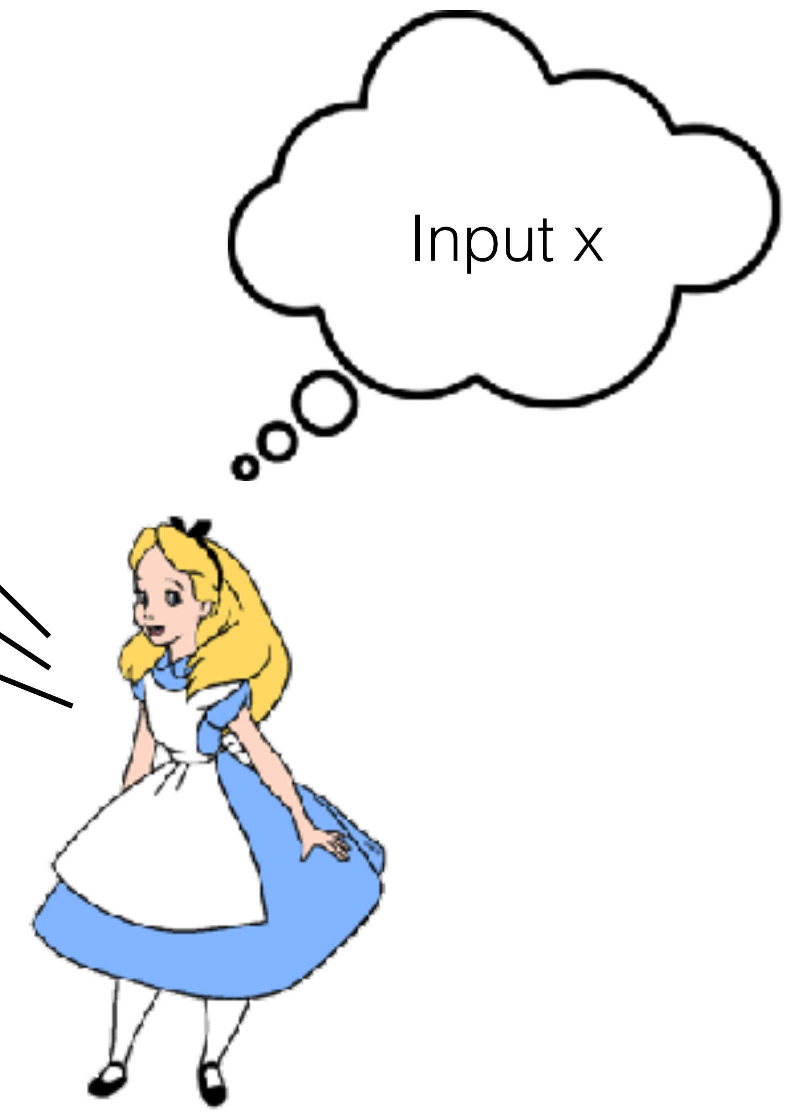

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$x \in \mathcal{L} \implies$ exists convincing proof

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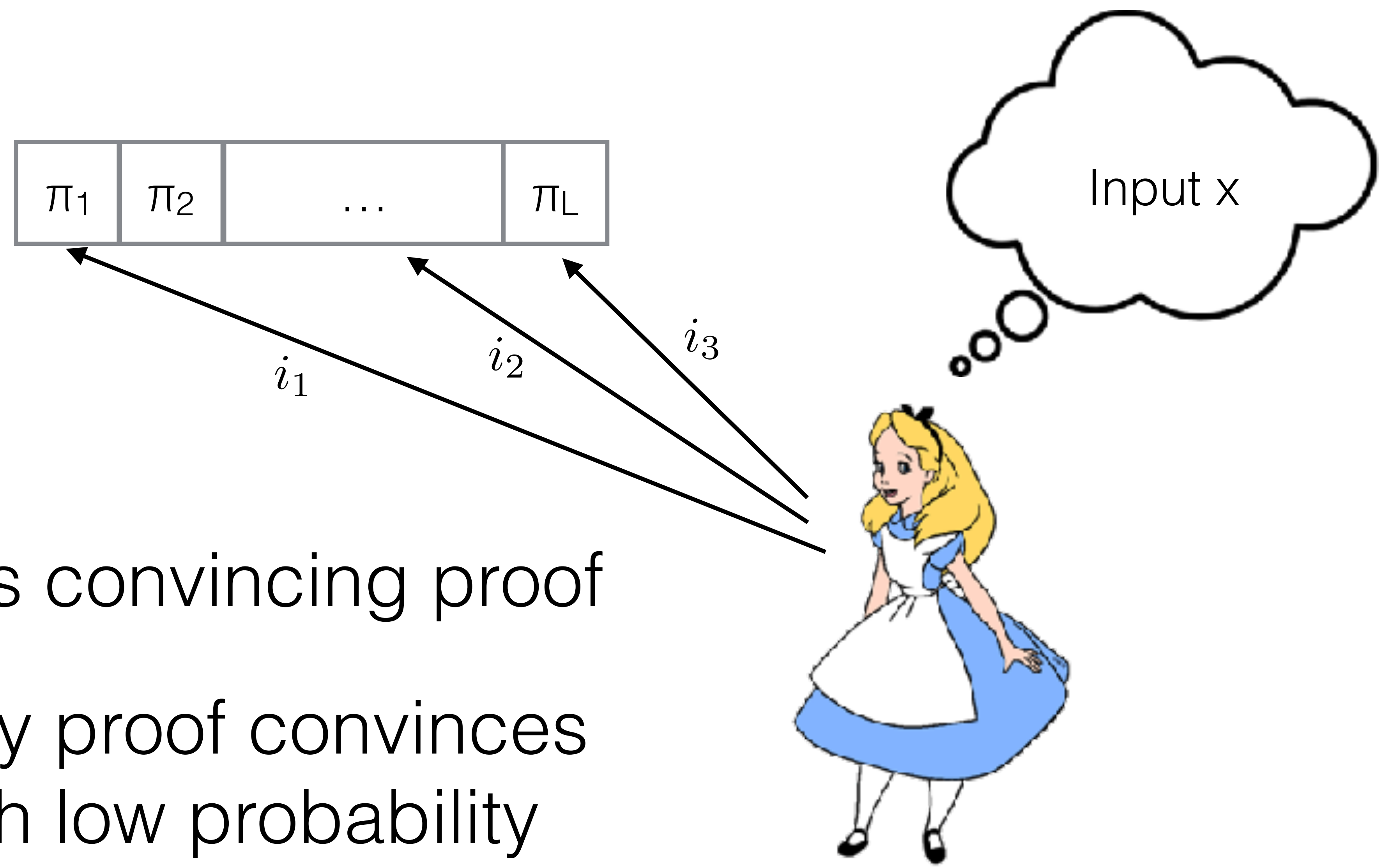

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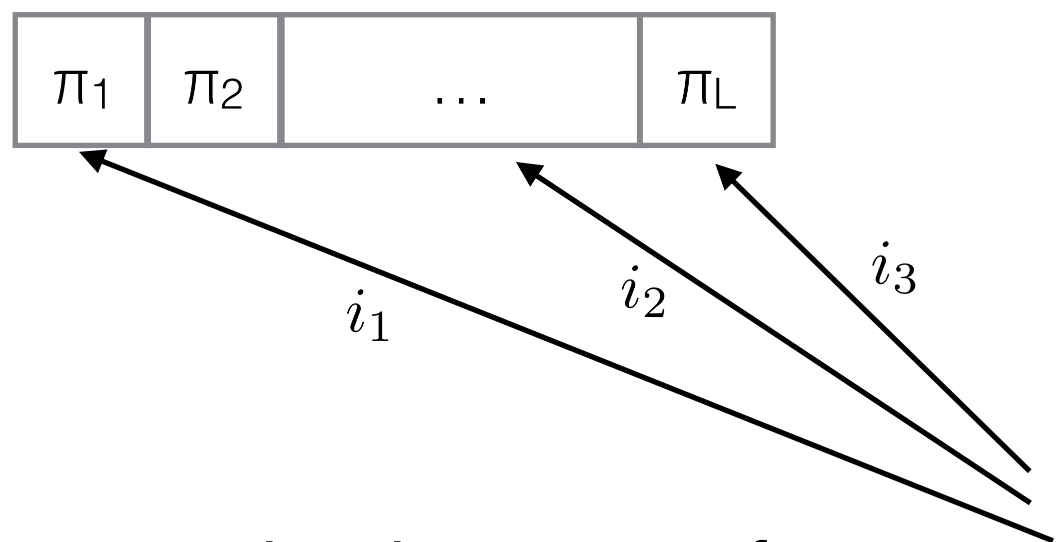

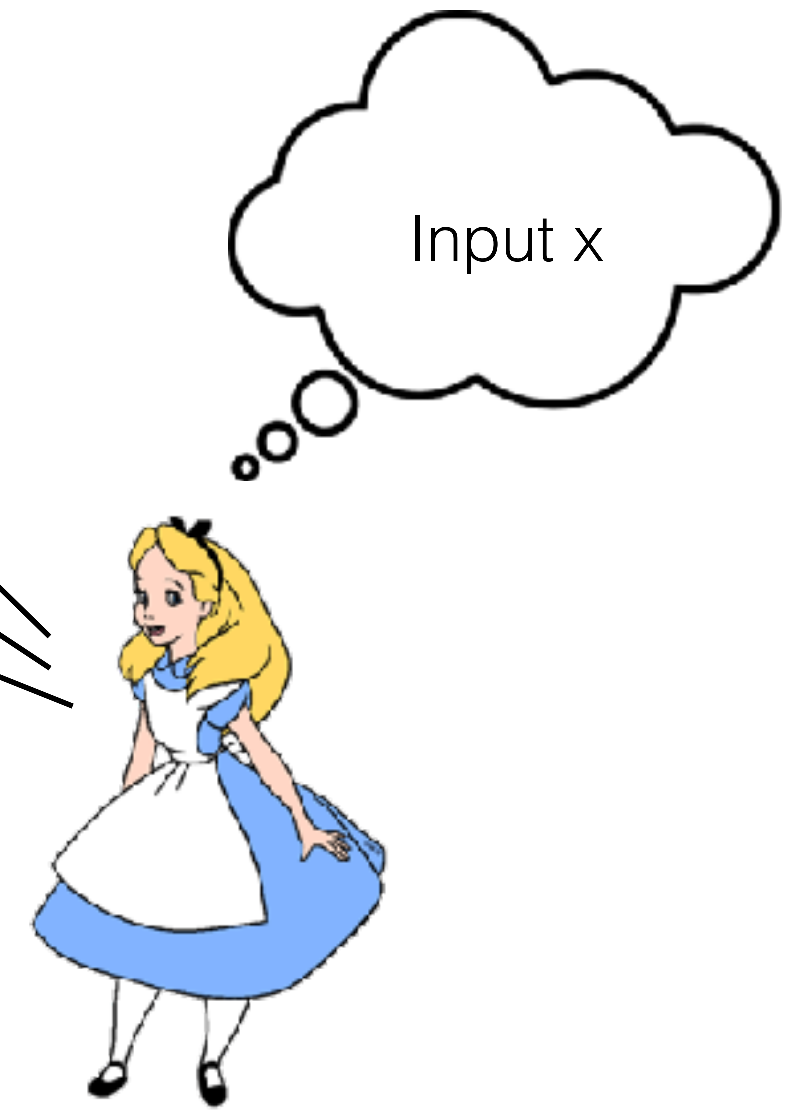

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Not a standard-model delegation scheme

PCP-based Delegation

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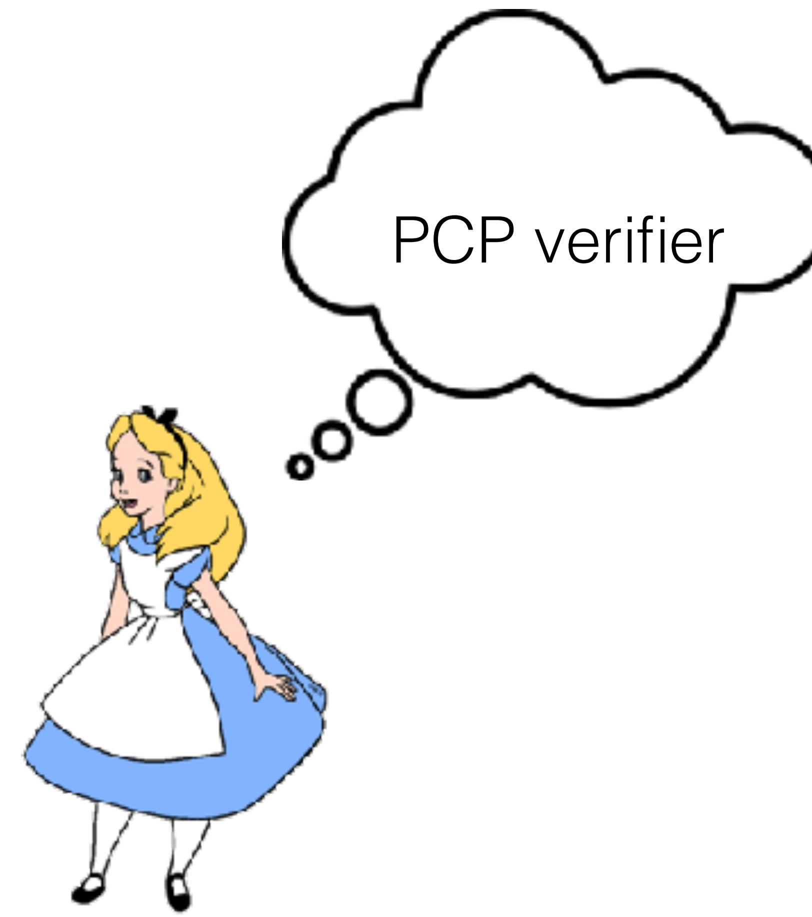

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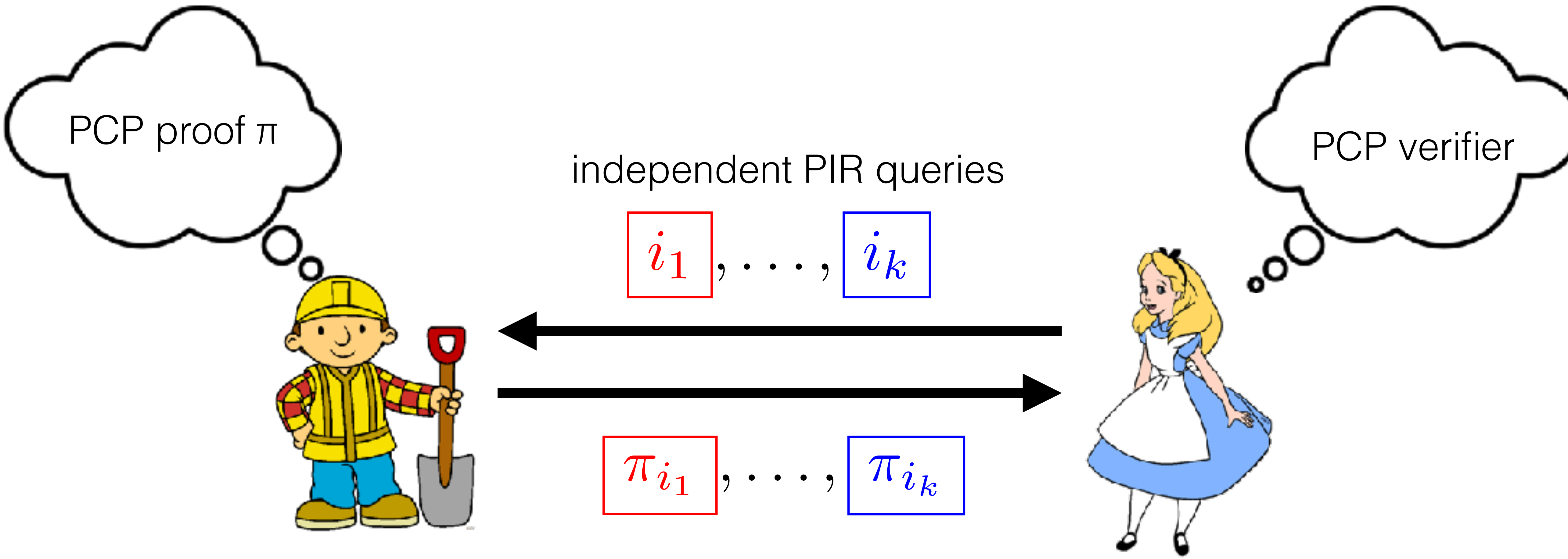

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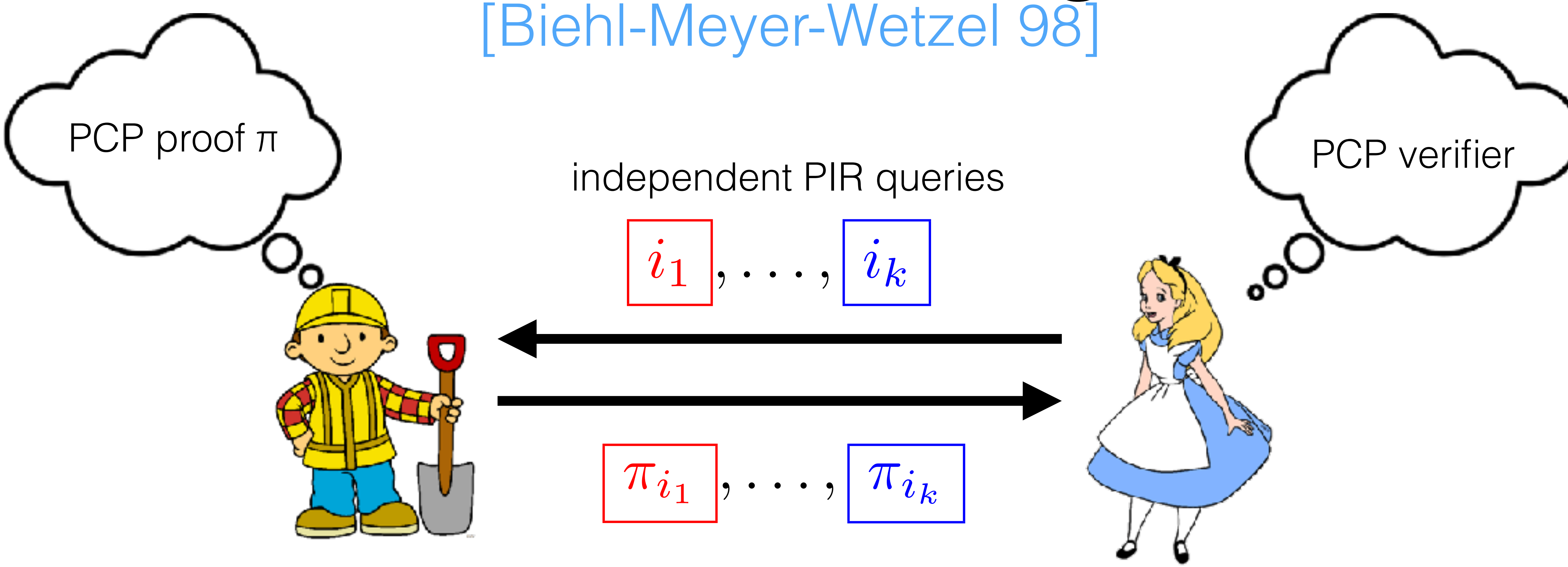
[Biehl-Meyer-Wetzel 98]



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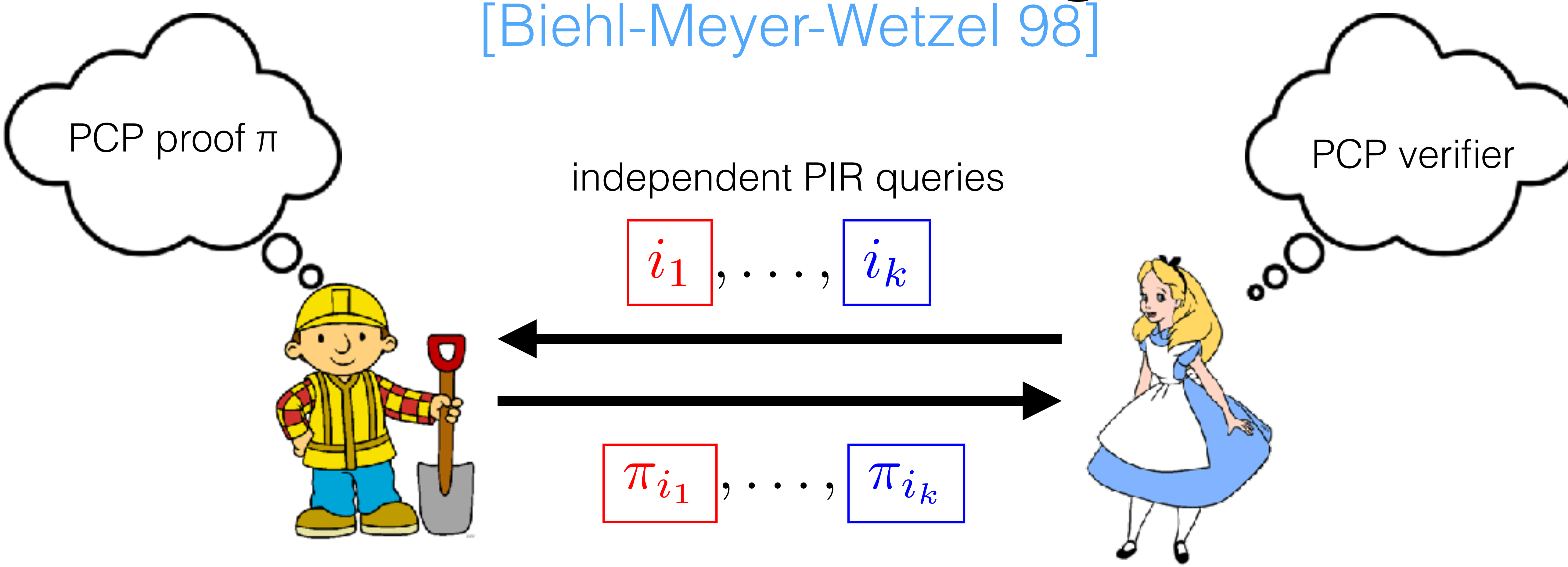

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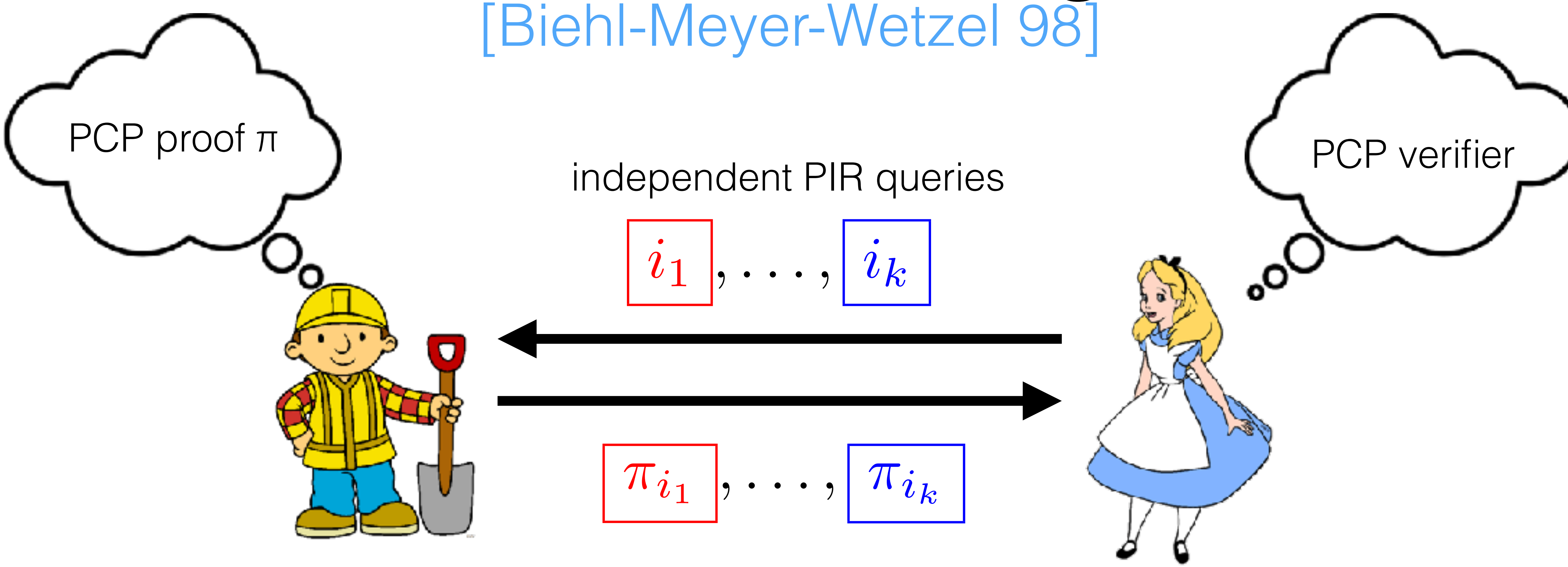

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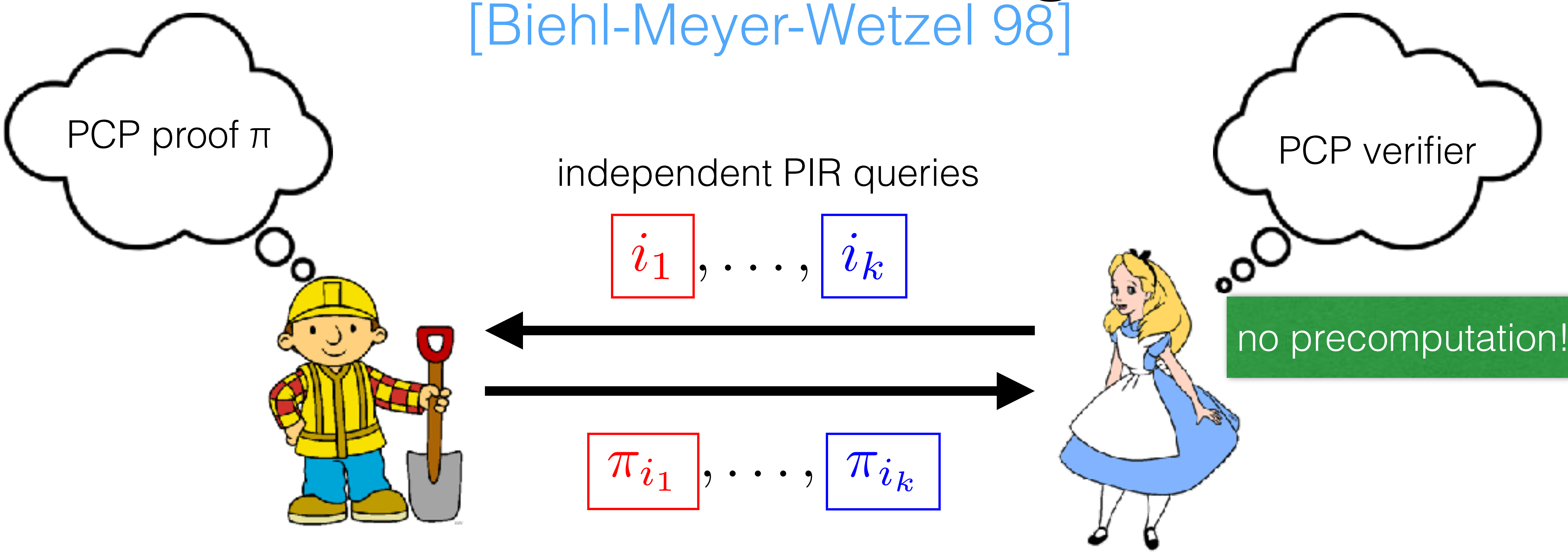

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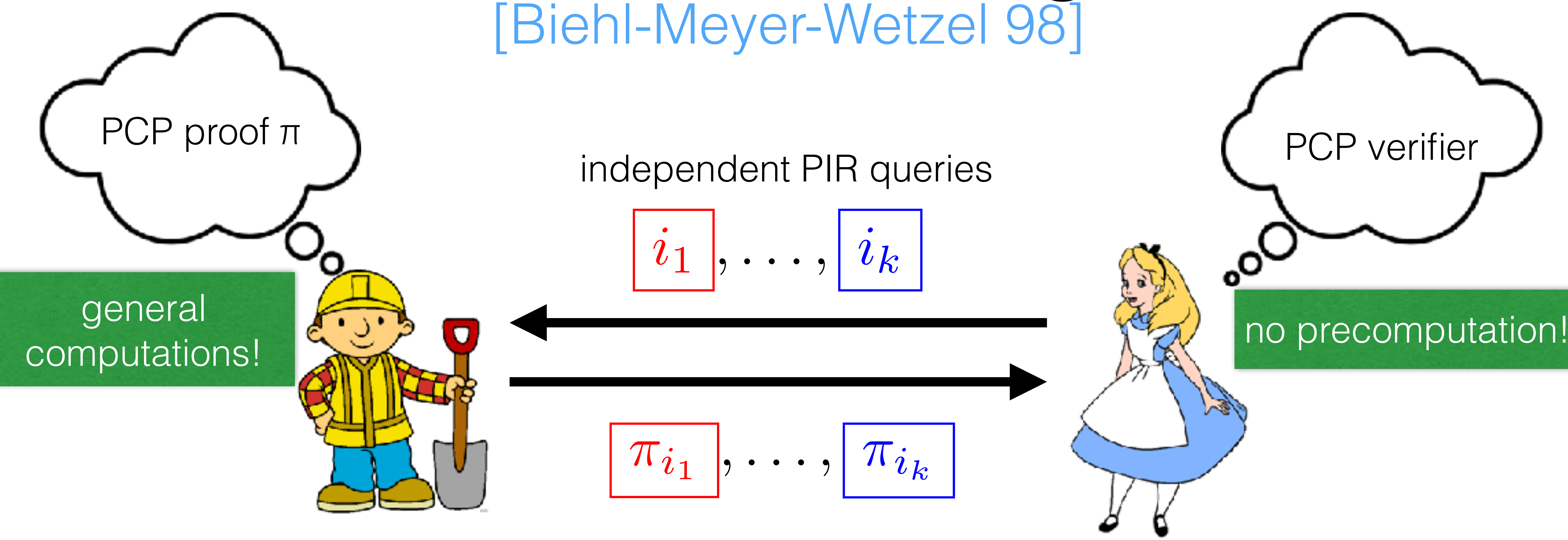

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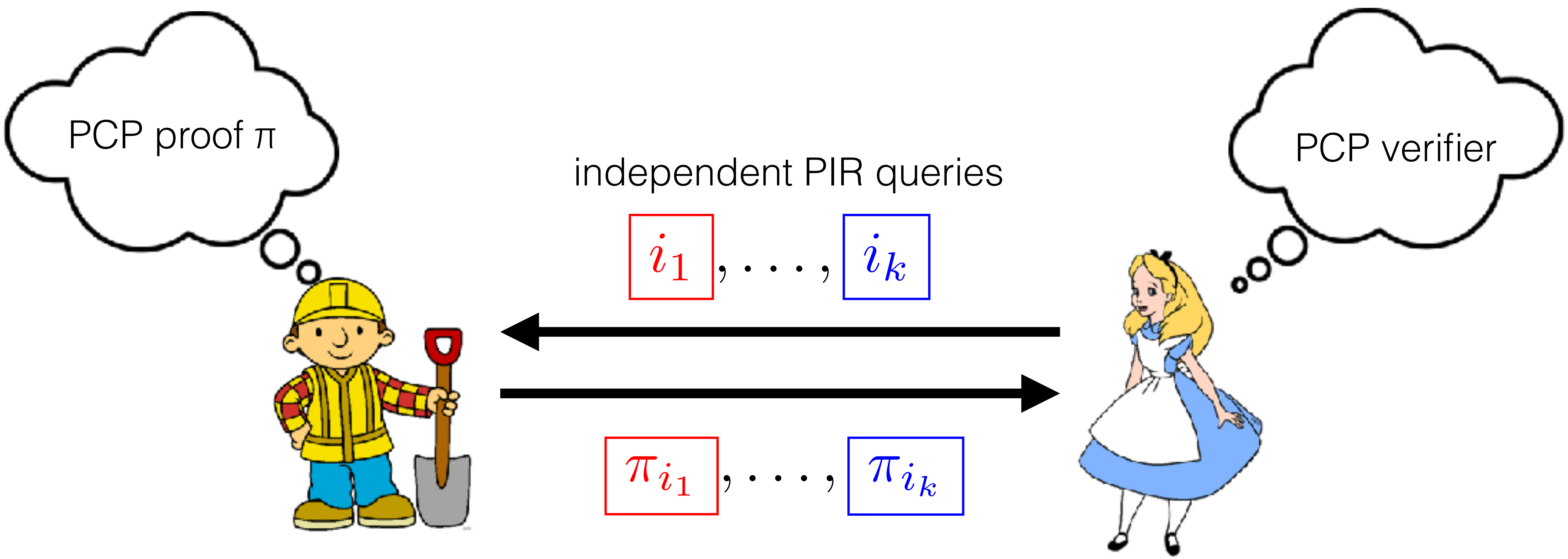

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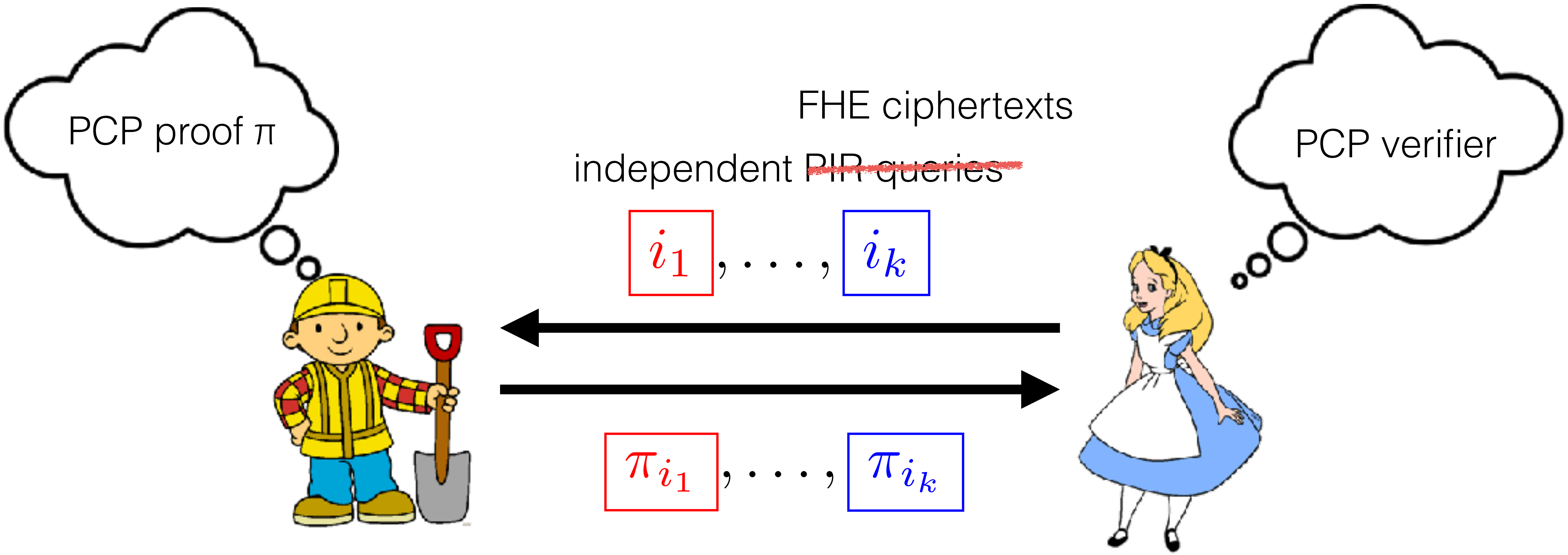

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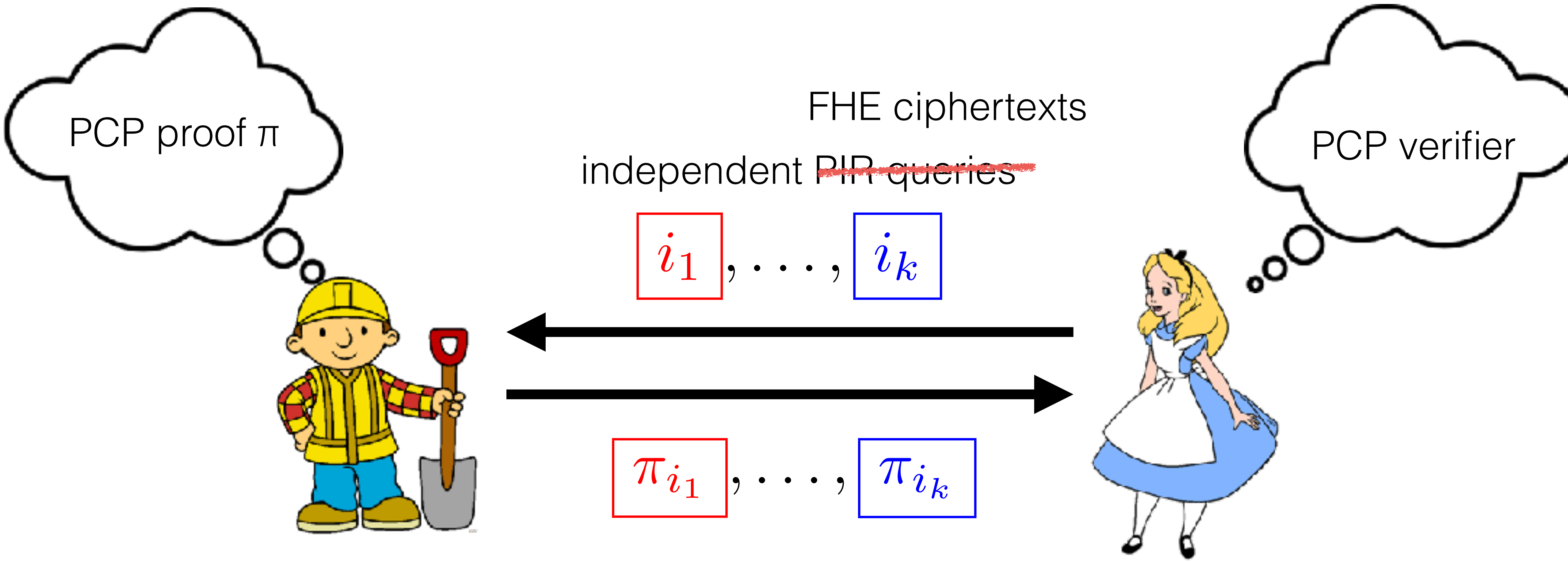

- If PIR = FHE, just need efficient “random-access” to PCP.

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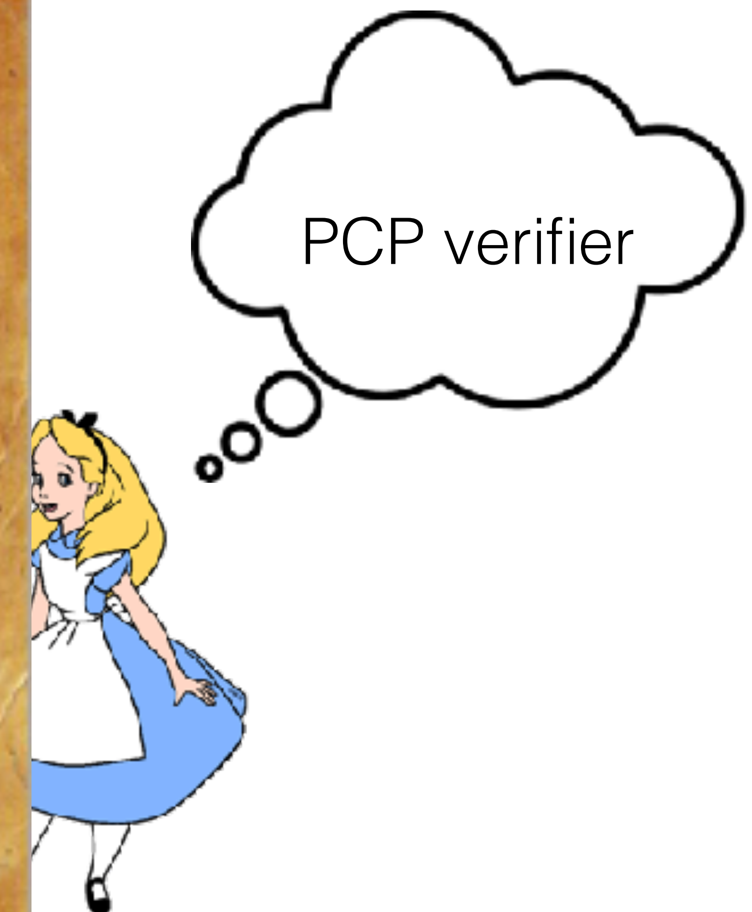

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  access” t…

Our Technical Contributions

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1. Simpler and direct NS-PCP(essentially BFLS) for any language $\mathcal{L} \in \mathsf{TISP}(T, S)$

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BFLS already known to be complexity-preserving? [BC12, BTVW14]

in

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for ***deterministic*** computations

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Talk Outline

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NOT proving NS-soundness of BFLS for deterministic circuits

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Part 1: Turing / RAM Machines ➡ (non-succinct) *deterministic* circuits

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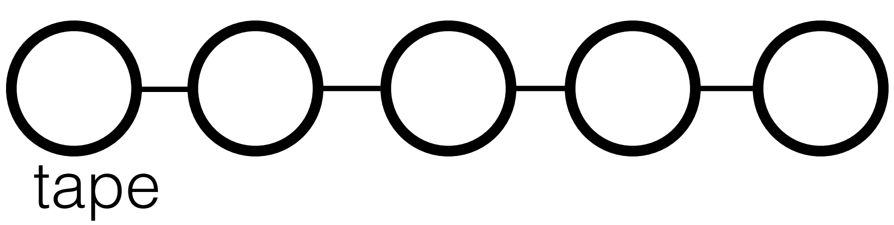
Part 1: Turing / RAM Machines ➡ (non-succinct) *deterministic* circuits

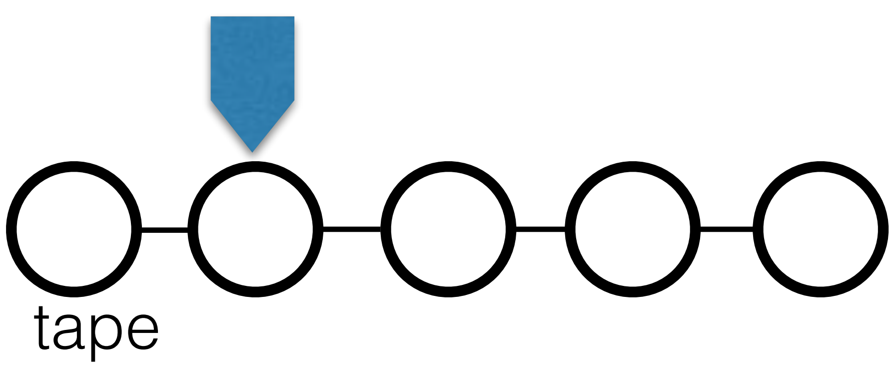
Part 2: (part of) BFLS prover efficiency despite non-succinctness.

Turing Machines as Circuits



TM Configuration

Turing Machines as Circuits



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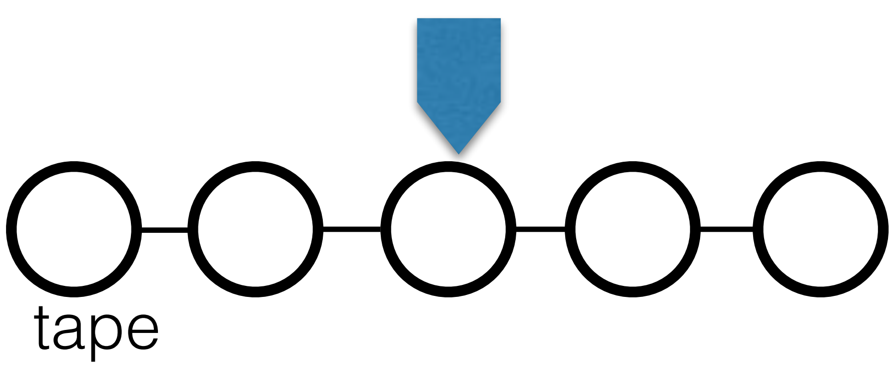

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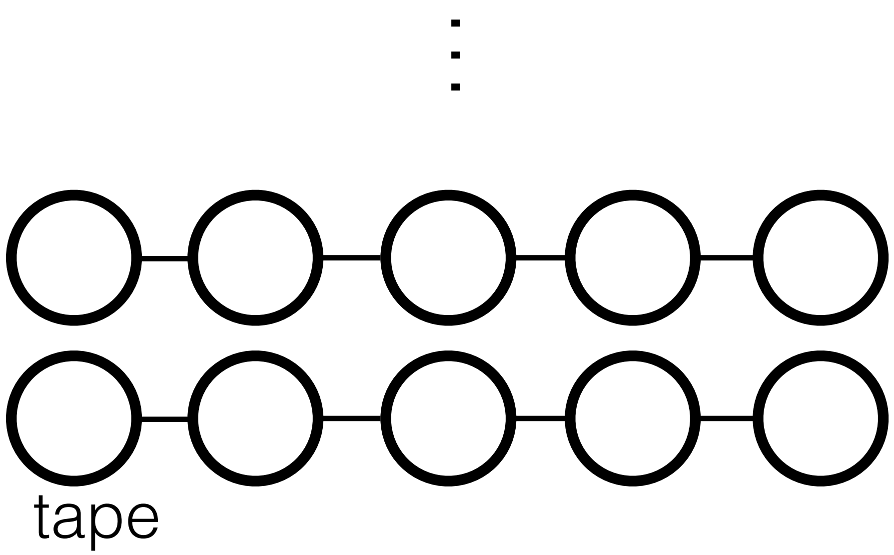

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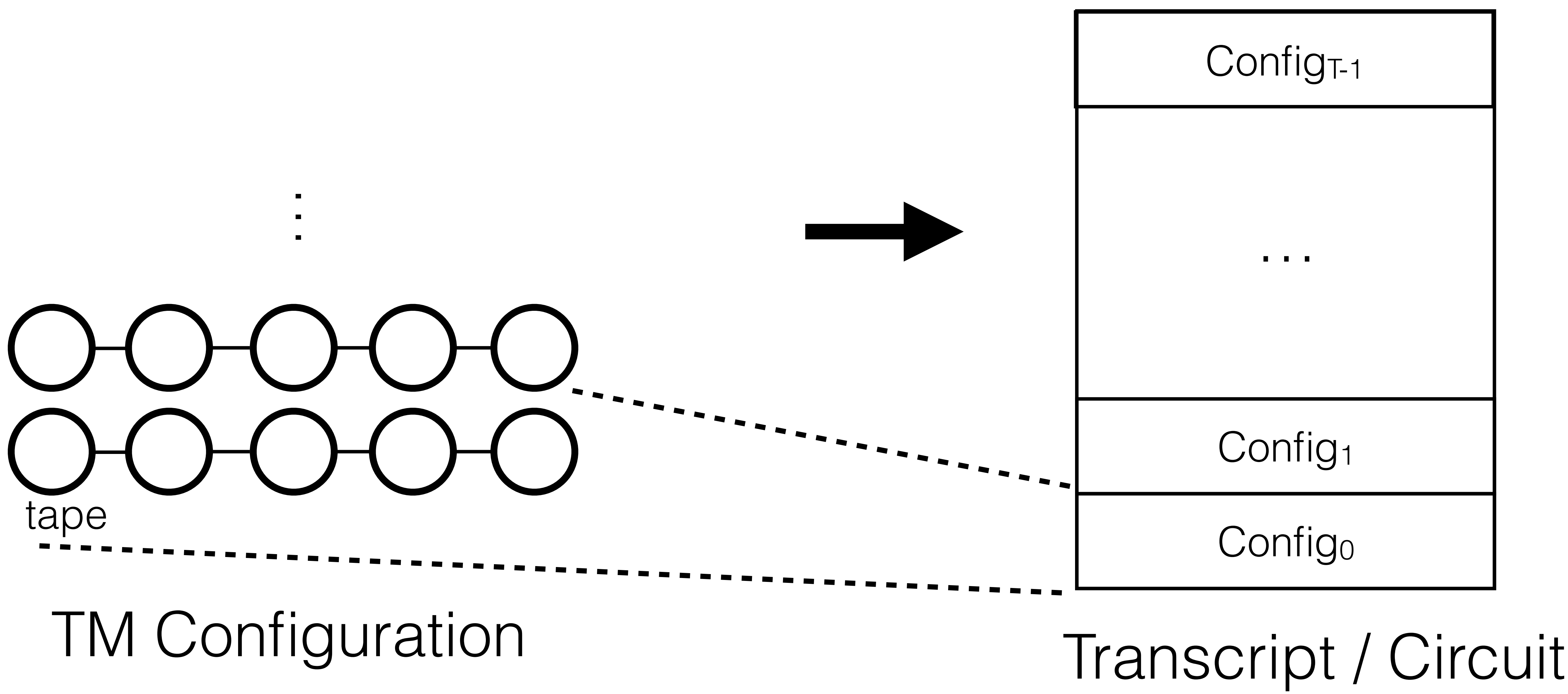

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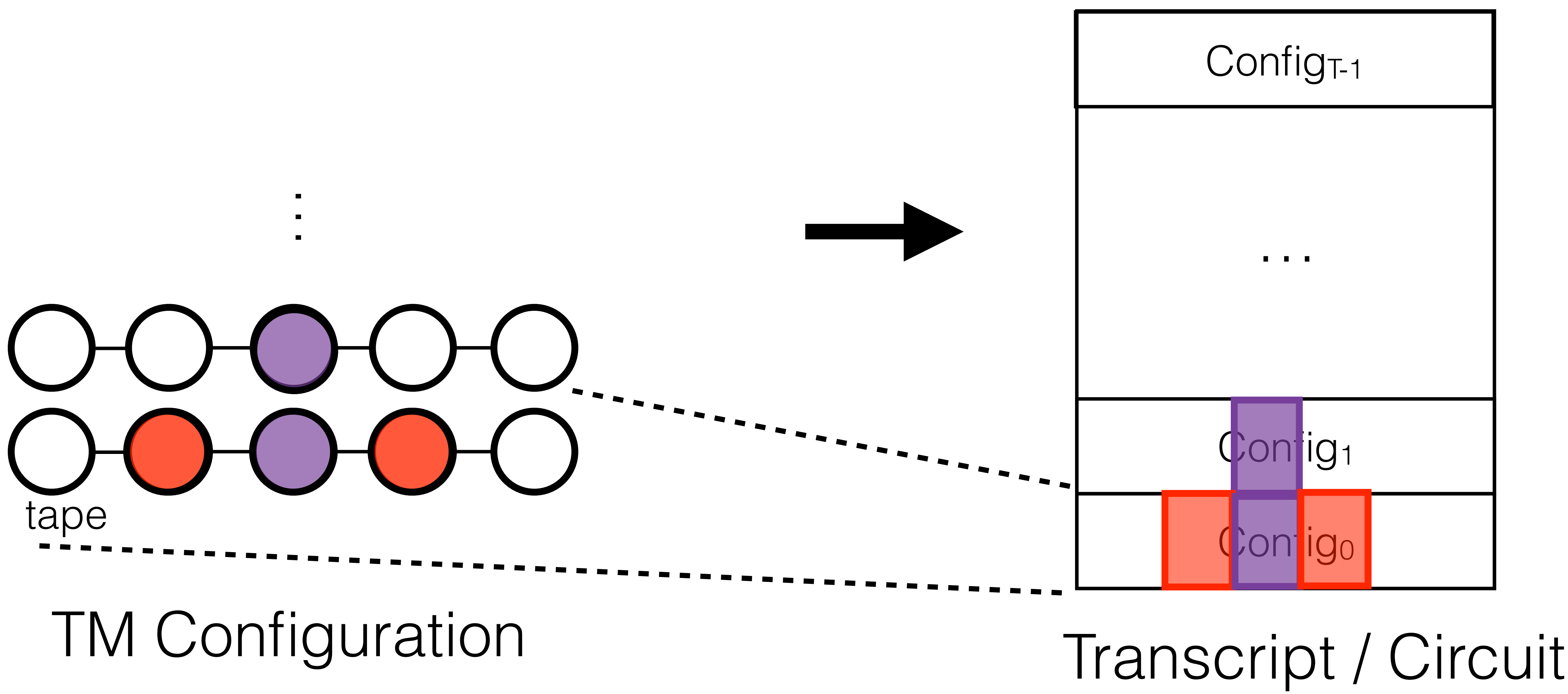

RAM Machines as Circuits

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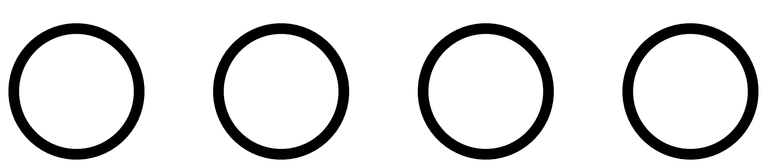

RAM Machines as Circuits

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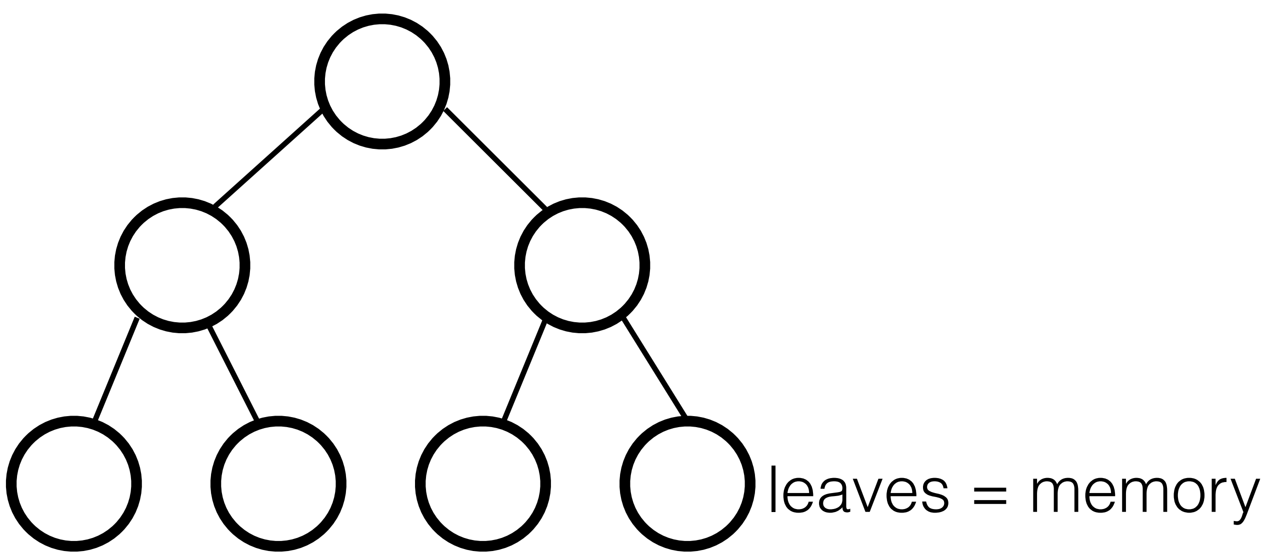
(diameter log S)



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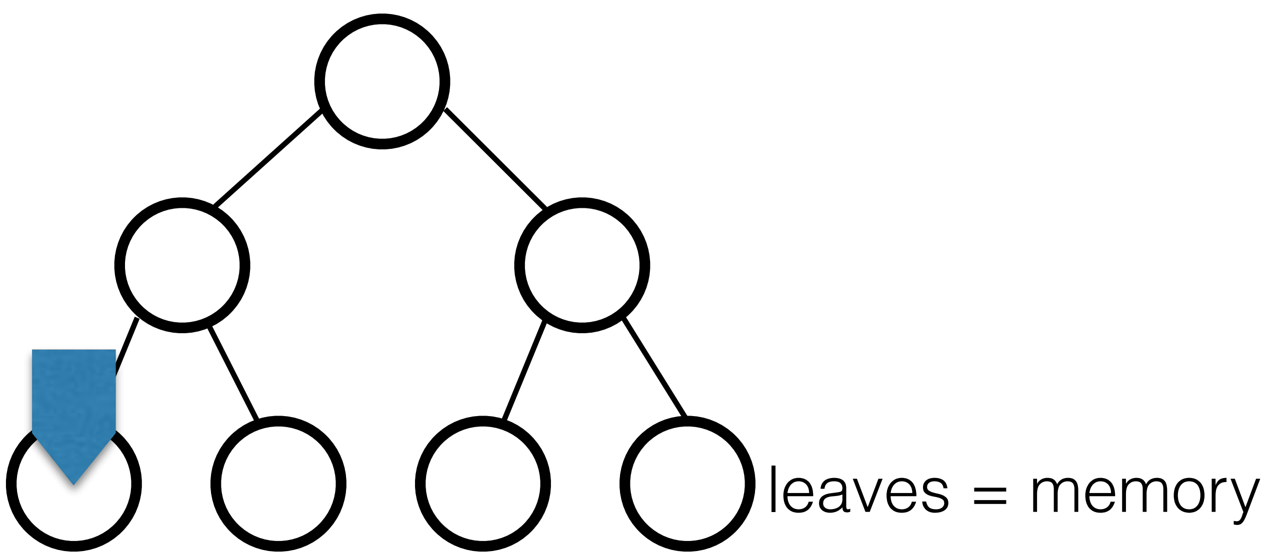

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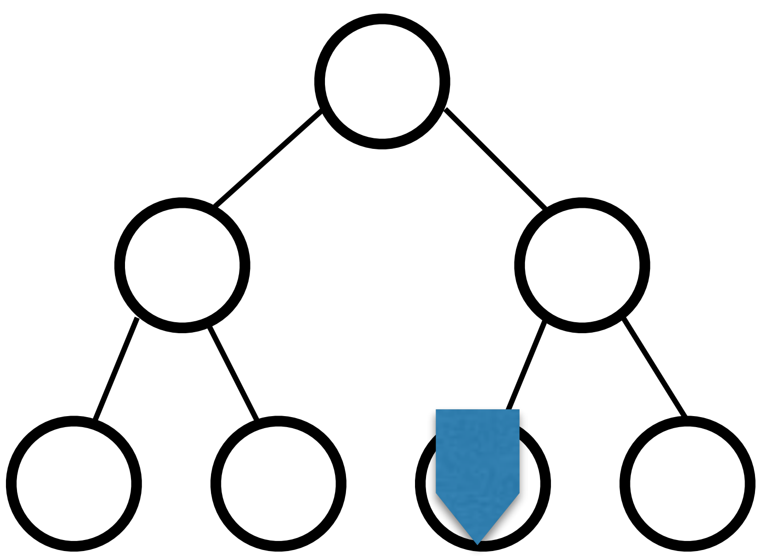

leaves = memory

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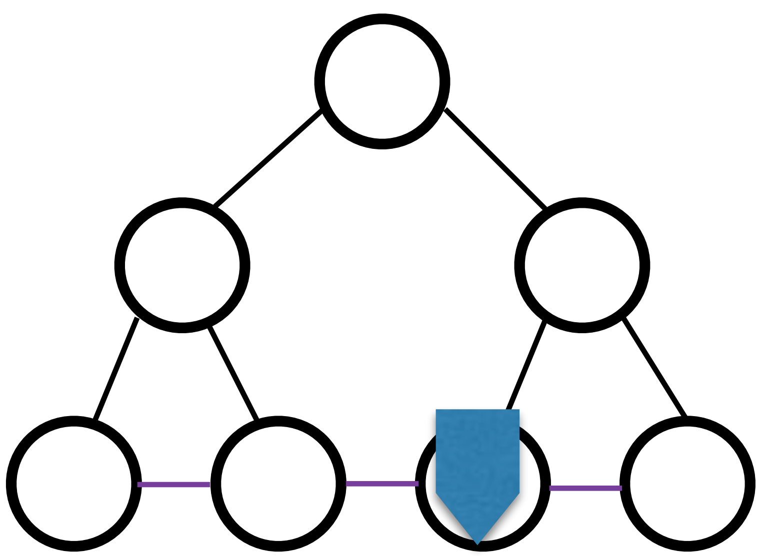

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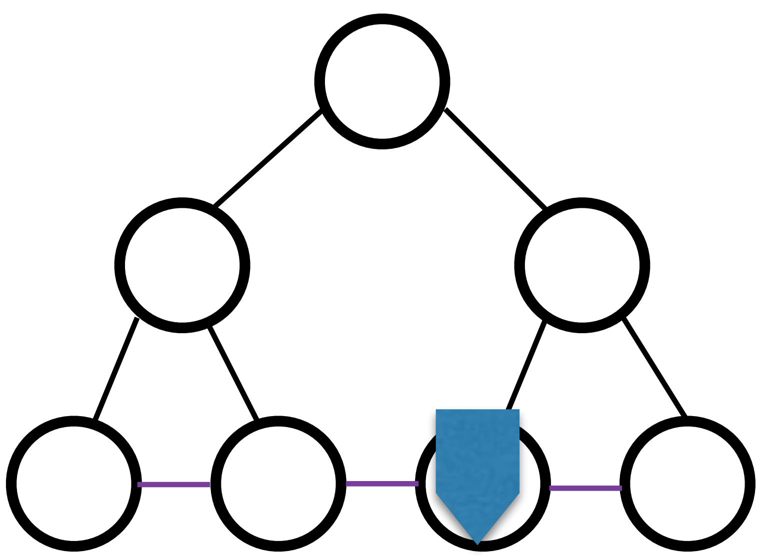

Important for BFLS:
Graph is “regular”!

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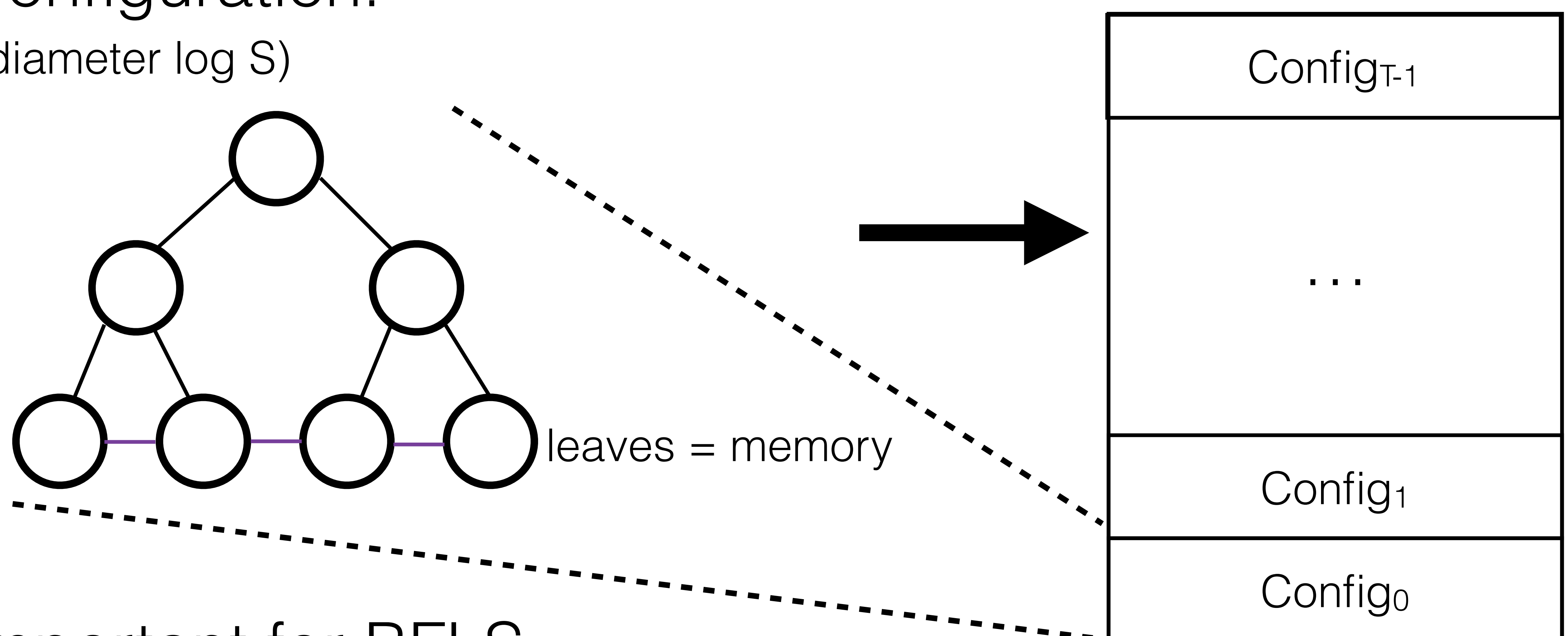

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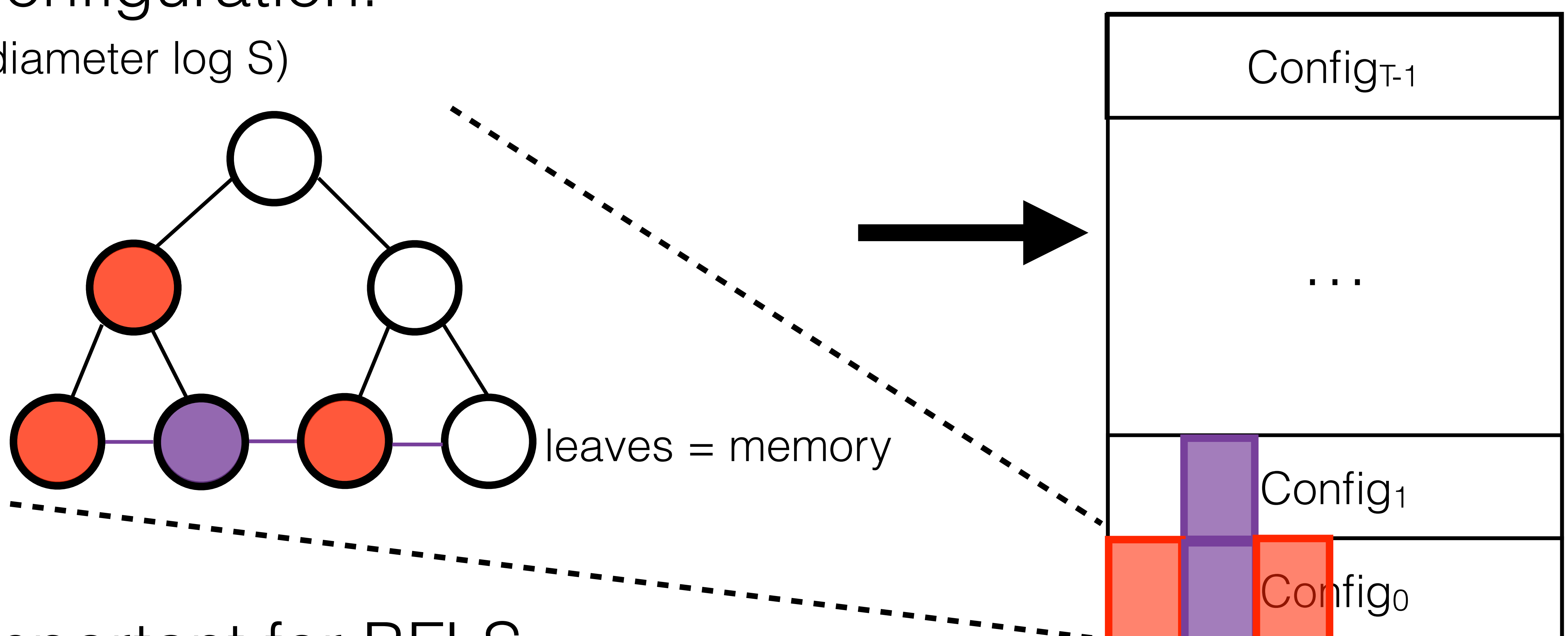

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RAM Machines as Circuits

no routing networks!

Transcript / Circuit:

Configuration:

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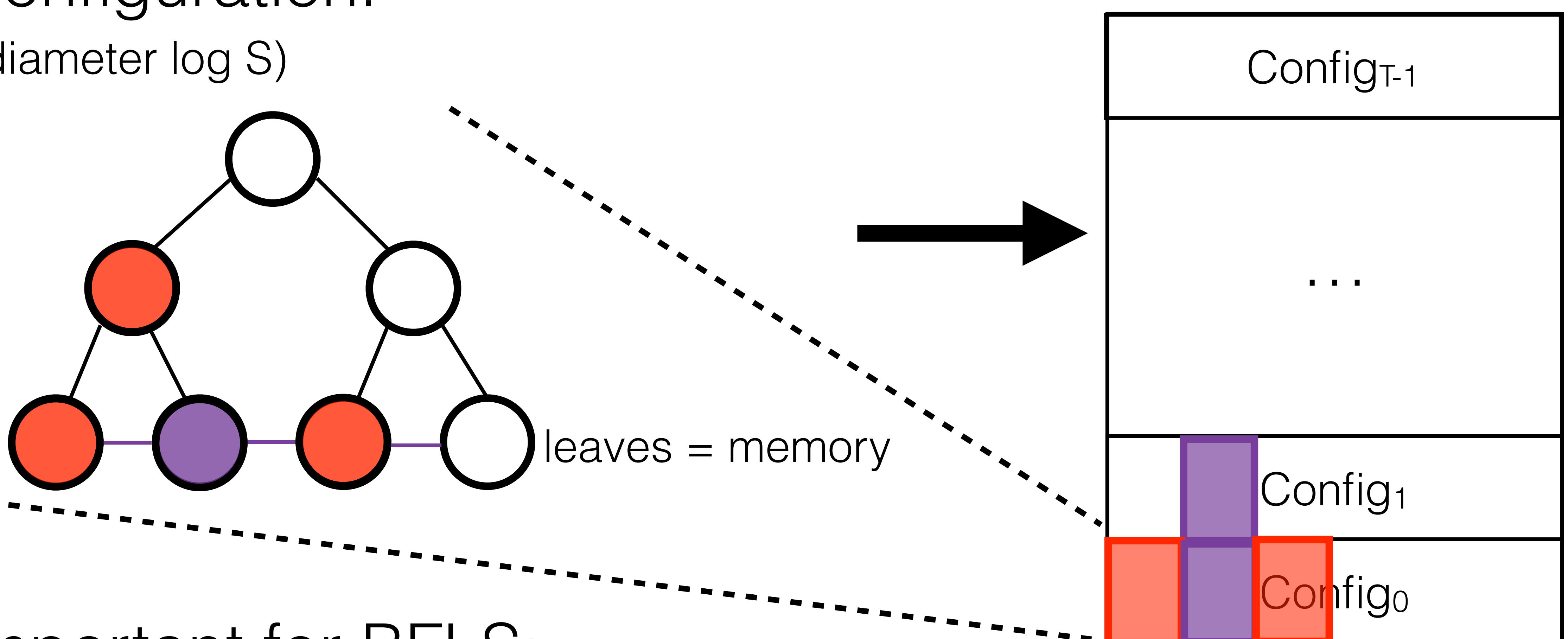

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no Merkle trees!

no routing networks!

Transcript / Circuit:

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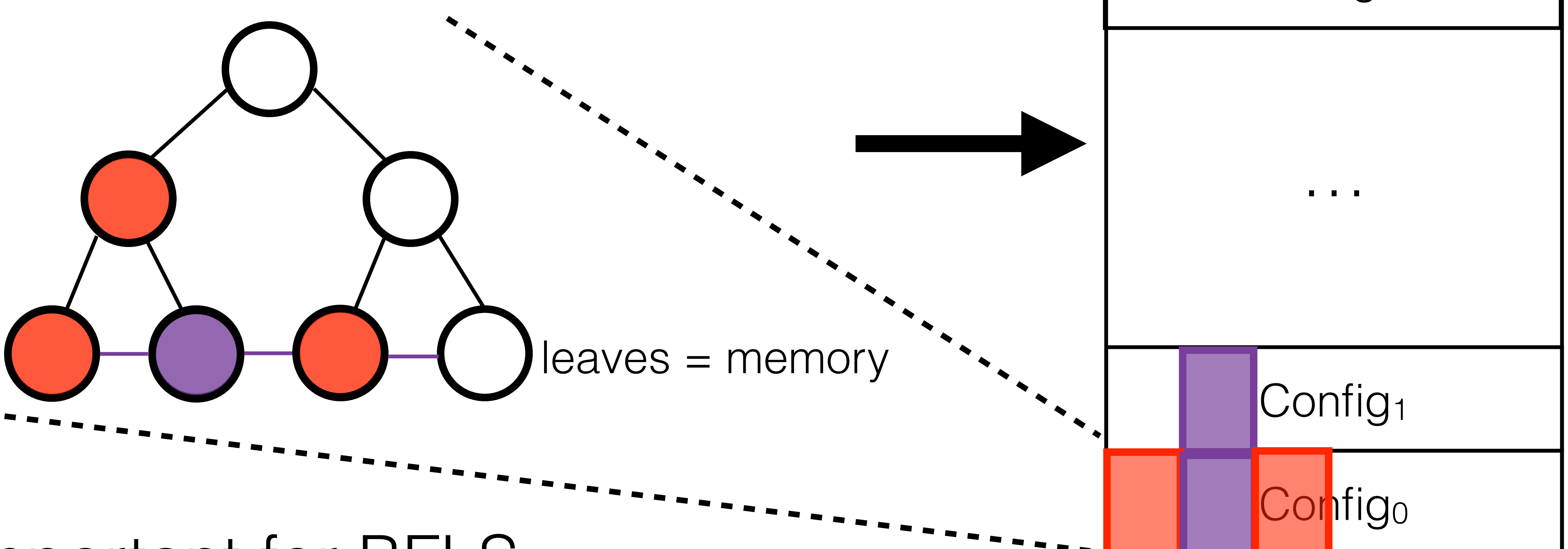

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0	0
0	1

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→

multilinear $\hat{f} : \mathbb{F}^m \to \mathbb{F}$

0	-2	-4	-6
0	-1	-2	-3
0	**0**	0	0
0	**1**	2	3

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$$\hat{f}(\mathbb{x}) = \sum_{\mathbf{x} \in \{0,1\}^m} f(\mathbf{x}) \cdot \hat{\mathbf{1}_{\mathbf{x}}}(\mathbb{x})$$

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0	**1**	2	3

"funny x" $\in \mathbb{F}^m$

$$\hat{f}(\mathbb{x}) = \sum_{\mathbf{x}\in\{0,1\}^m} f(\mathbf{x}) \cdot \hat{\mathbf{1}_{\mathbf{x}}}(\mathbb{x})$$

The PCP (BFLS) Part 1: Multilinear extension

Let $f : \{0,1\}^m \to \mathbb{F}$ be any function.

0	0
0	1

multilinear $\hat{f} : \mathbb{F}^m \to \mathbb{F}$

0	-2	-4	-6
0	-1	-2	-3
0	**0**	0	0
0	**1**	2	3

"funny x" $\in \mathbb{F}^m$

$$\hat{f}(\mathbb{x}) = \sum_{\mathbf{x} \in \{0,1\}^m} f(\mathbf{x}) \cdot \hat{\mathbf{1}_{\mathbf{x}}}(\mathbb{x})$$

"bold x" $\in \{0,1\}^m$

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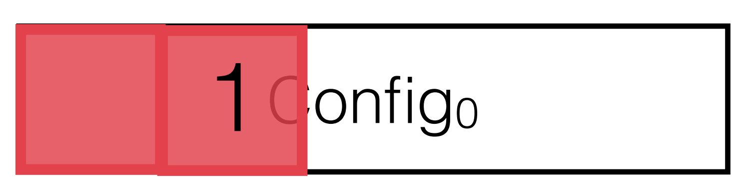

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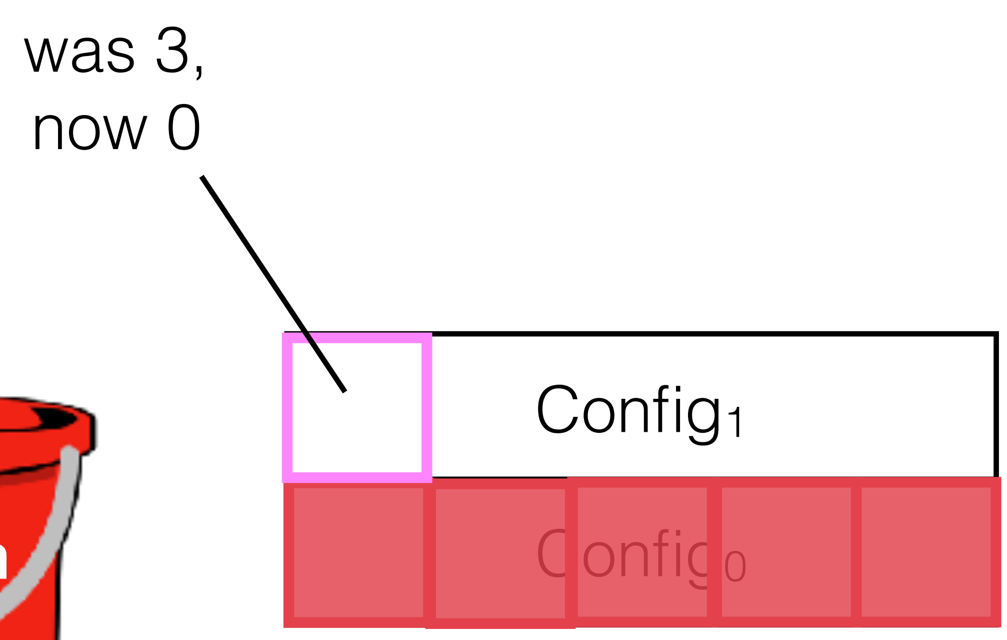

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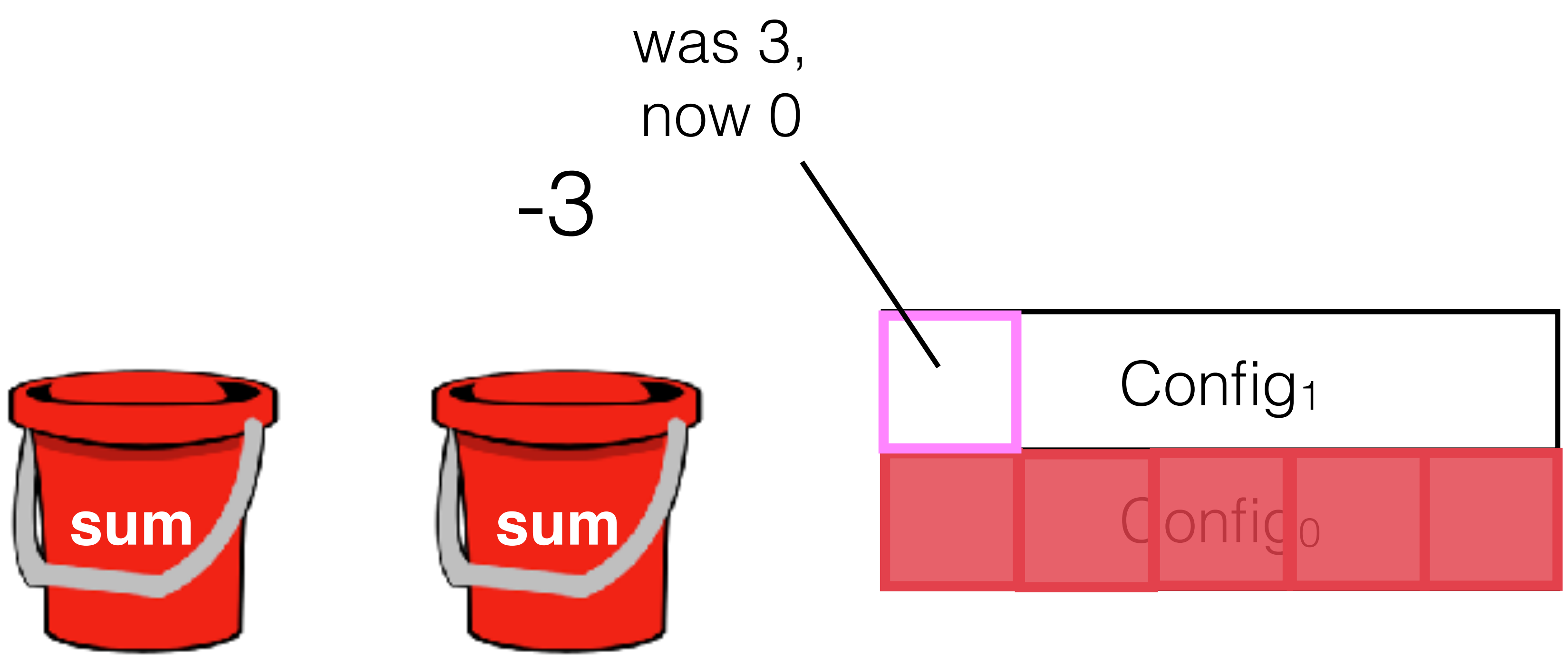

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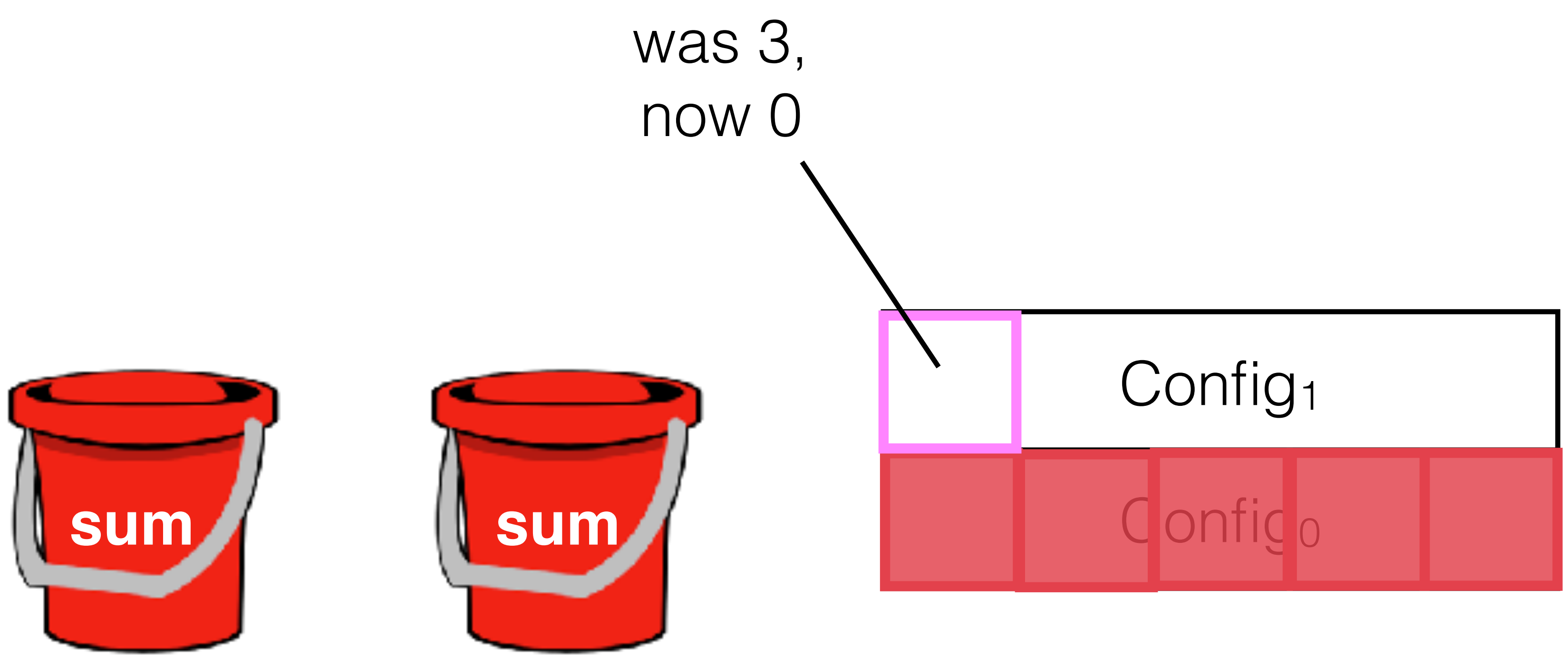

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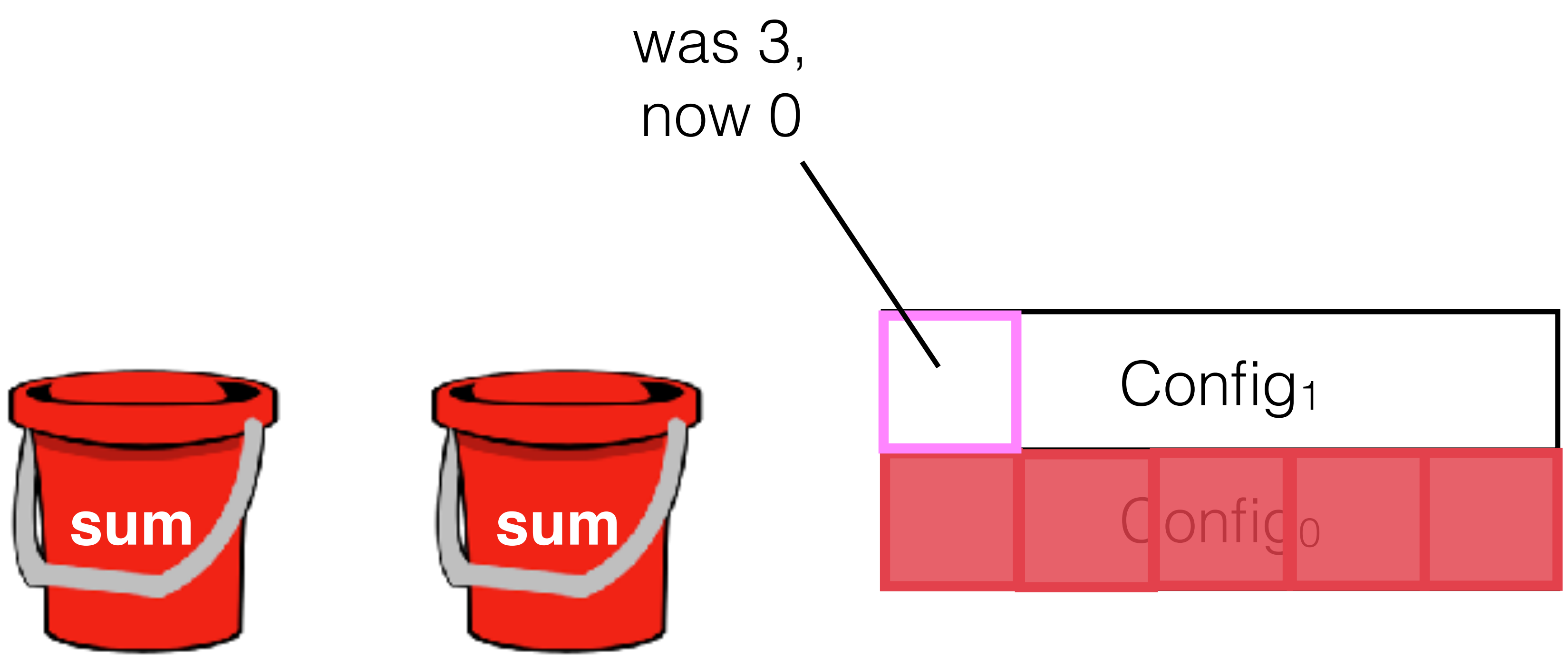

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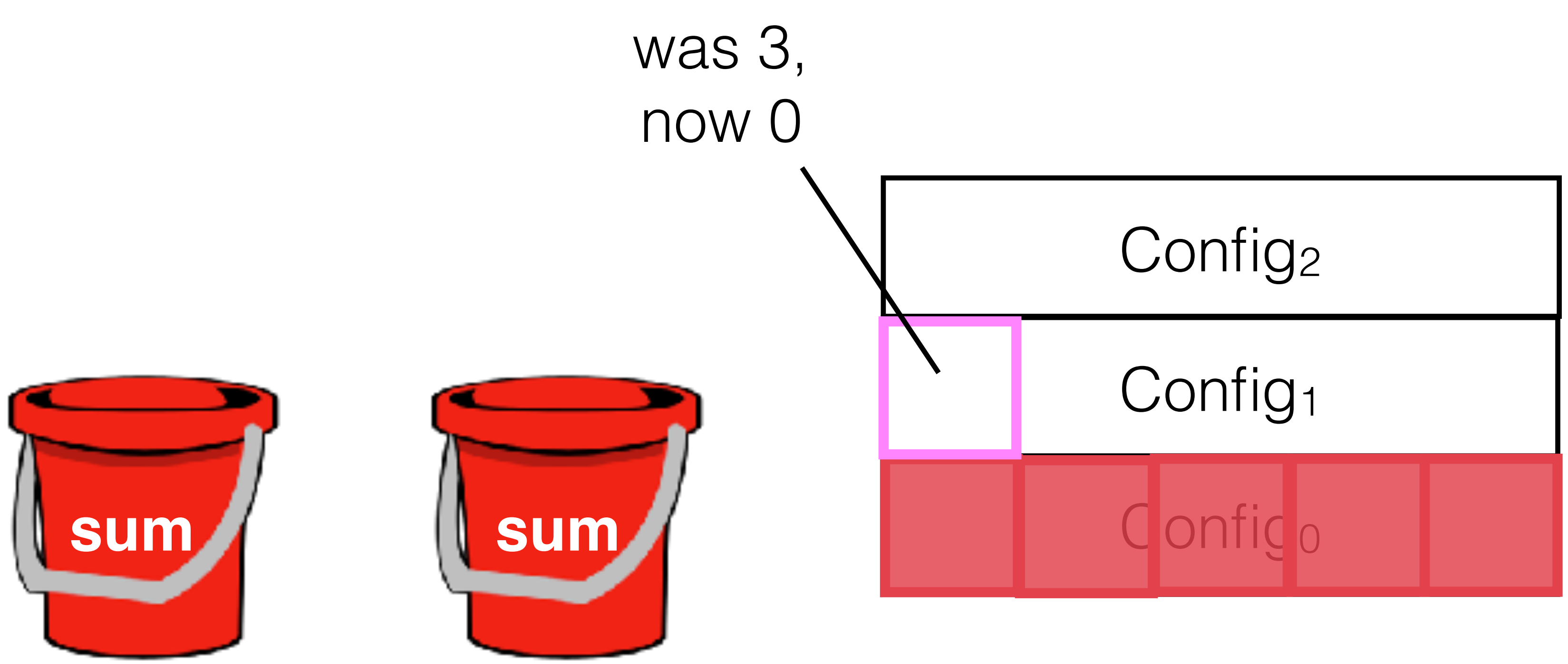

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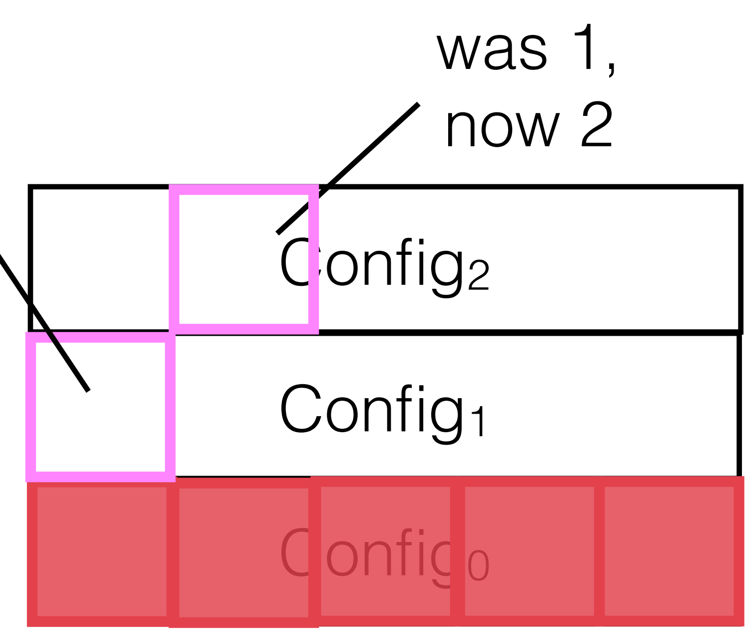

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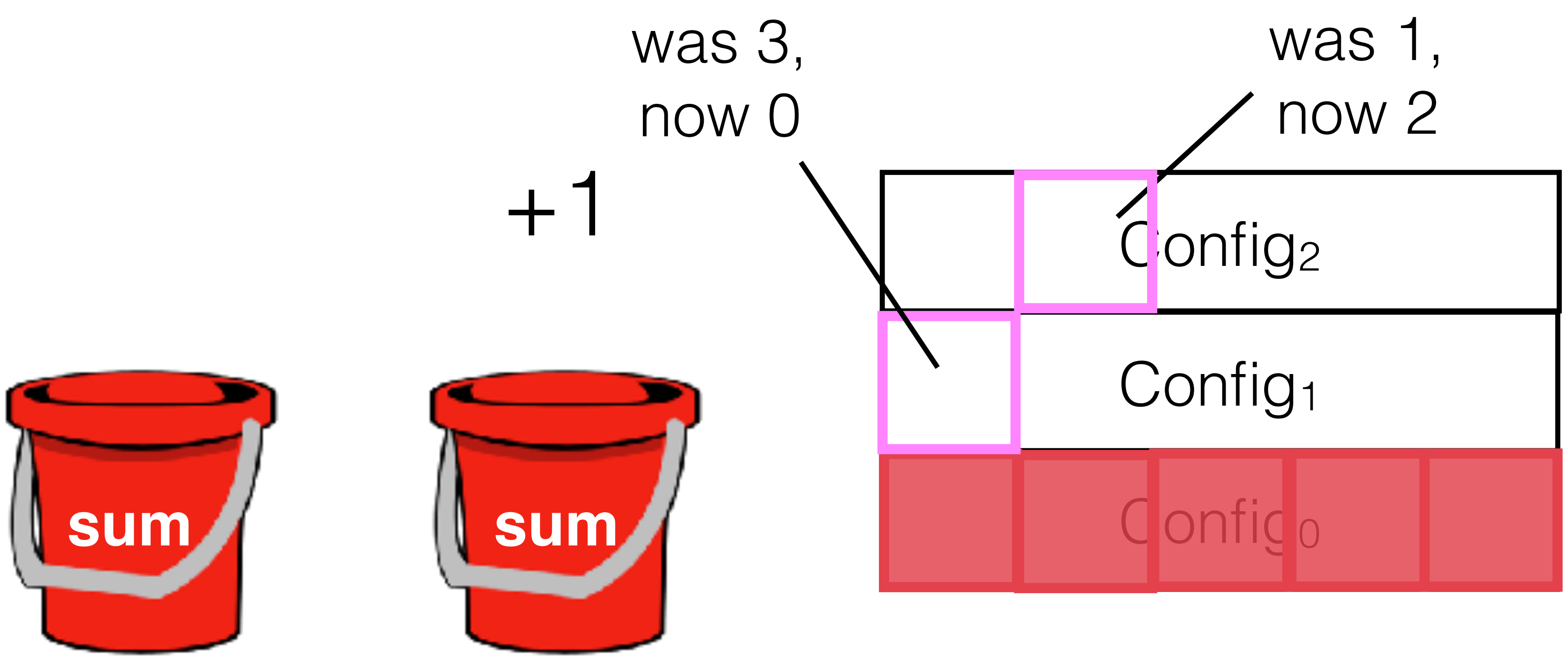

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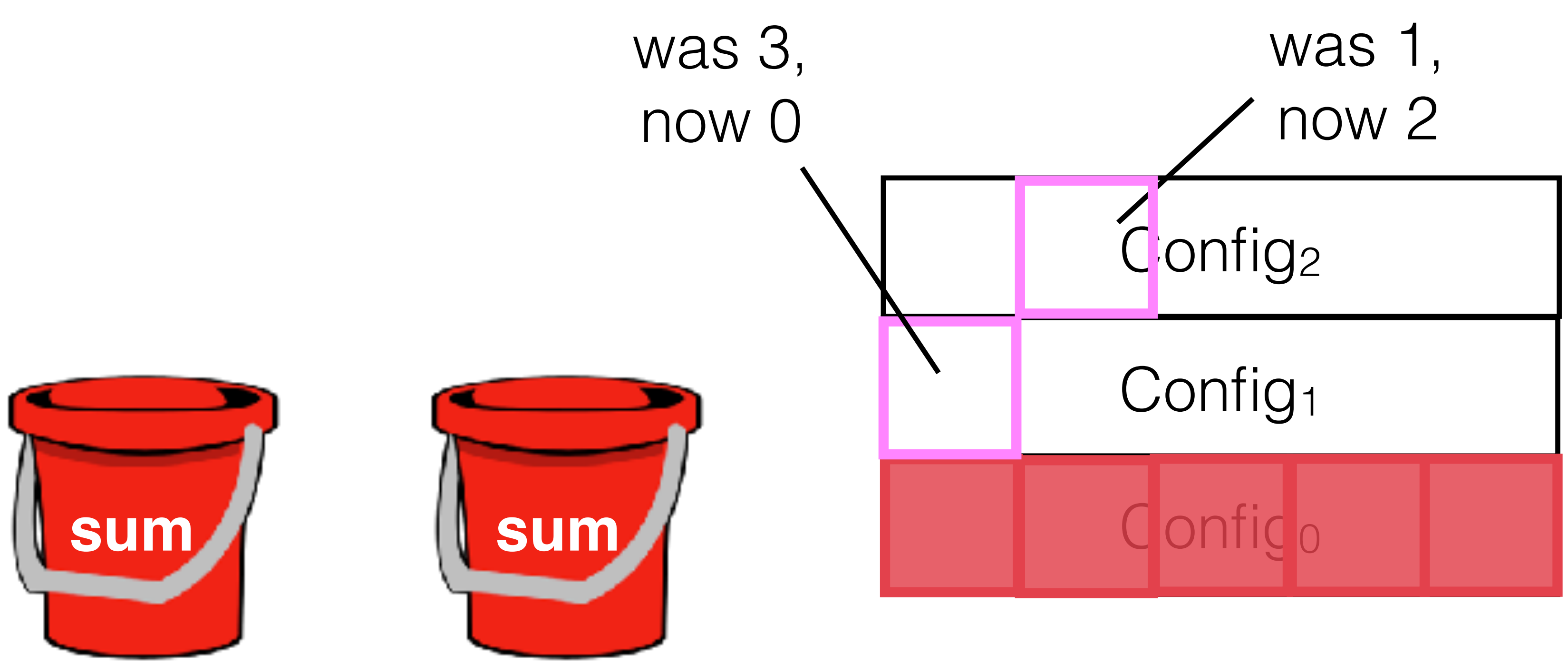

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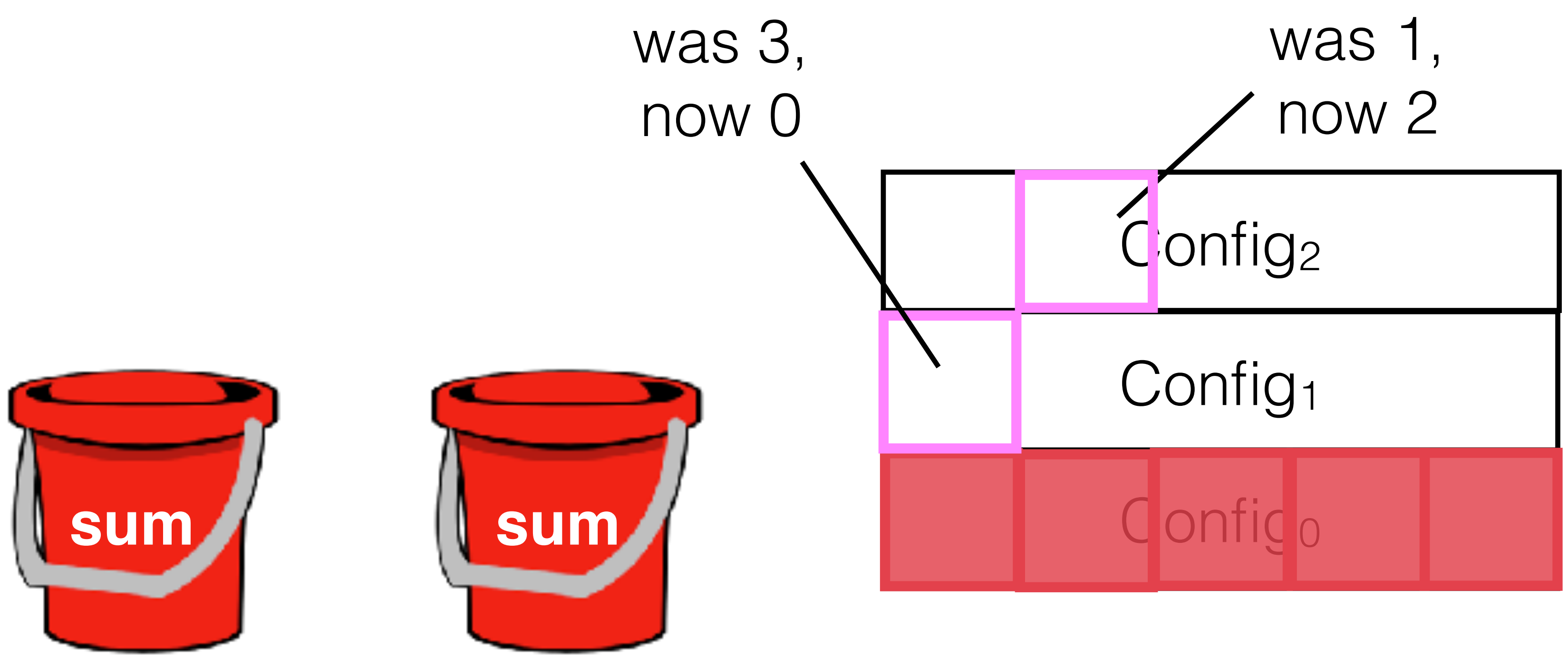

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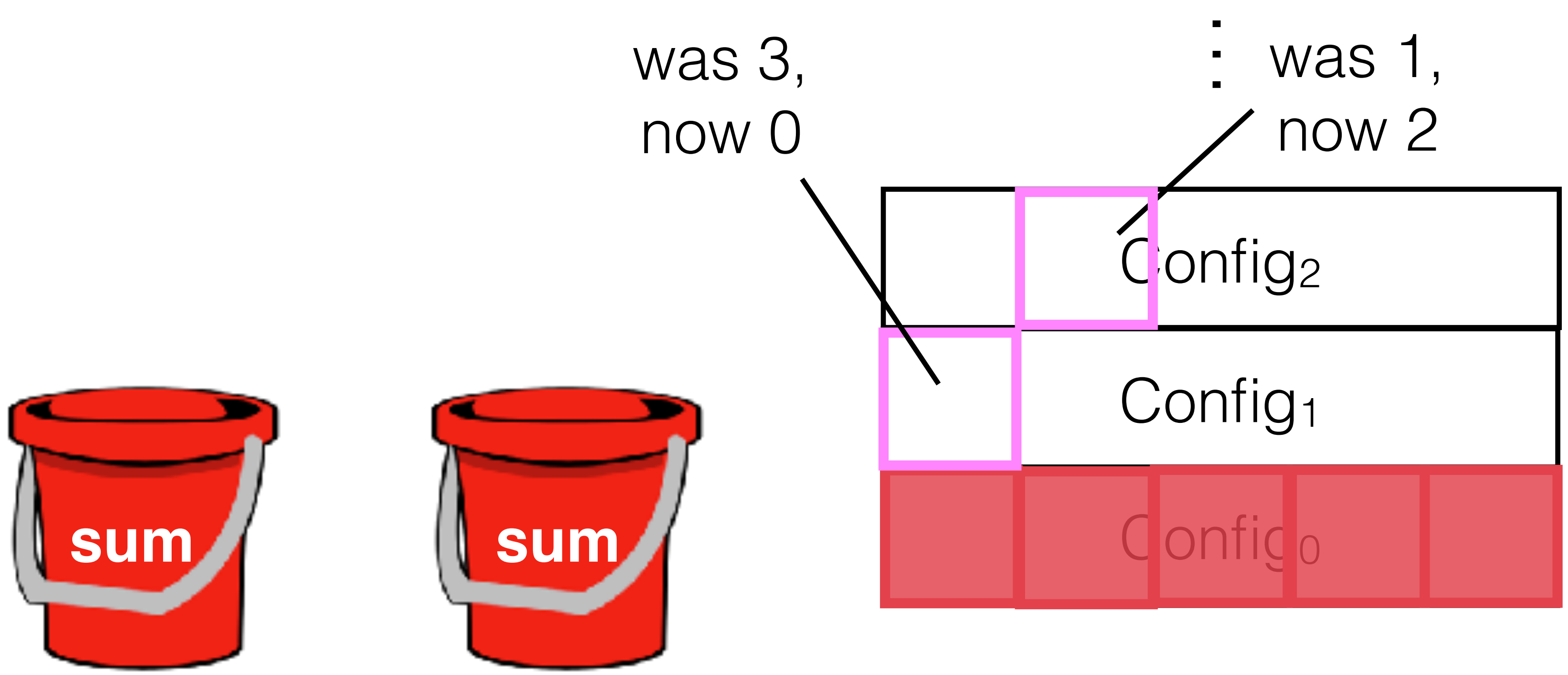

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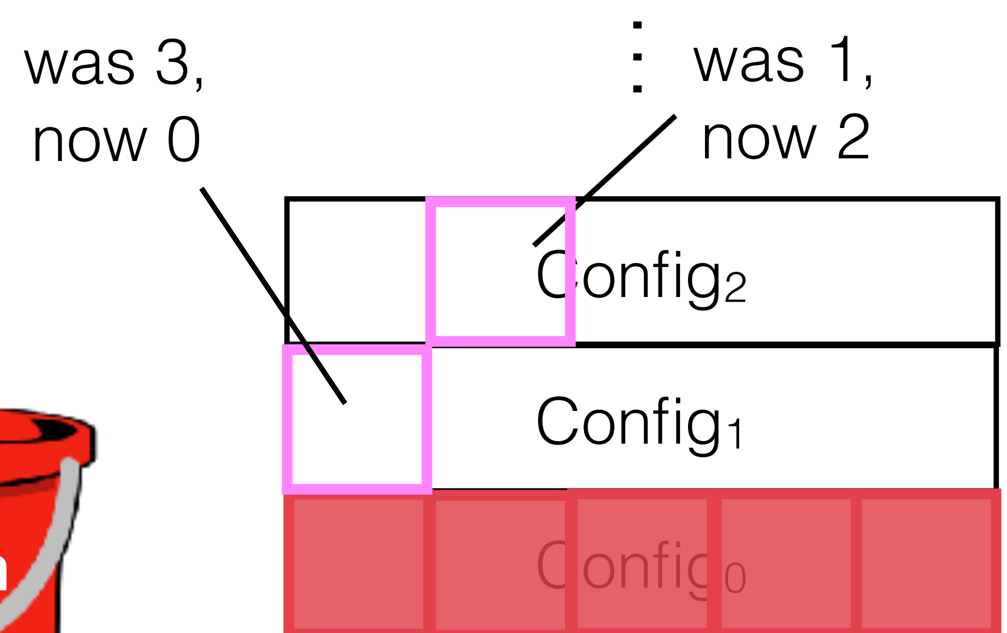

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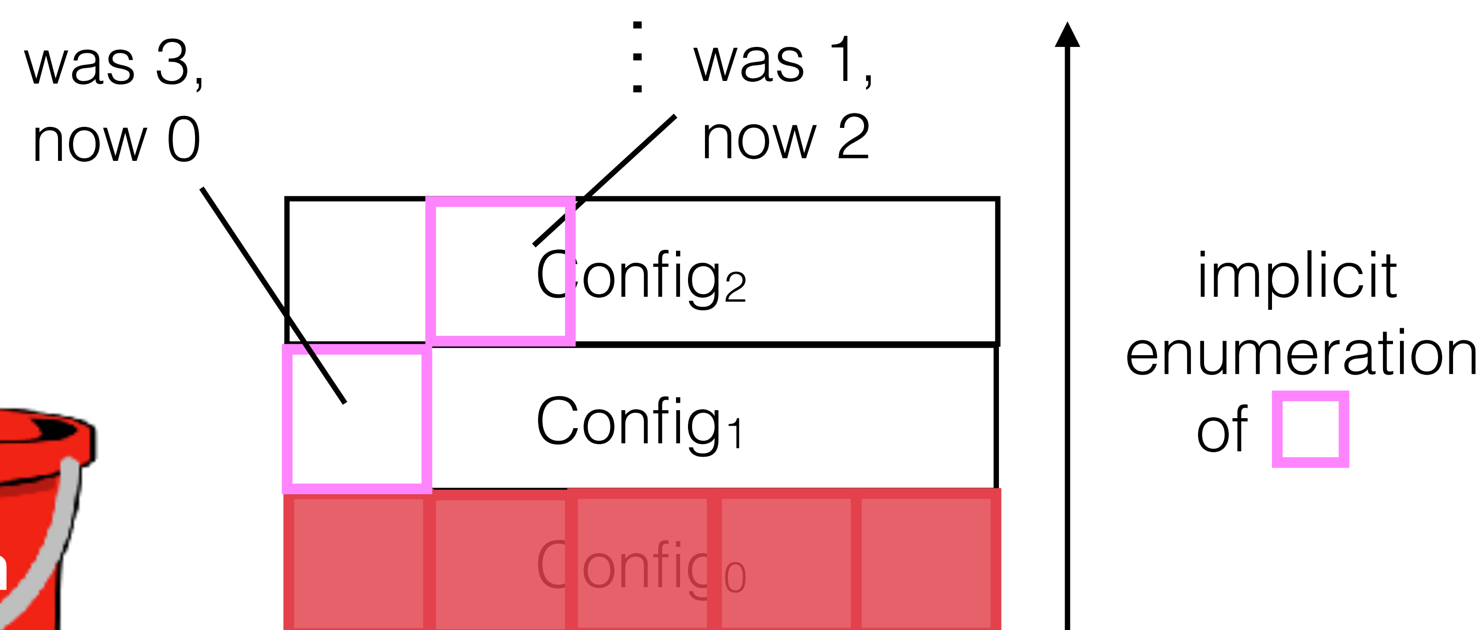

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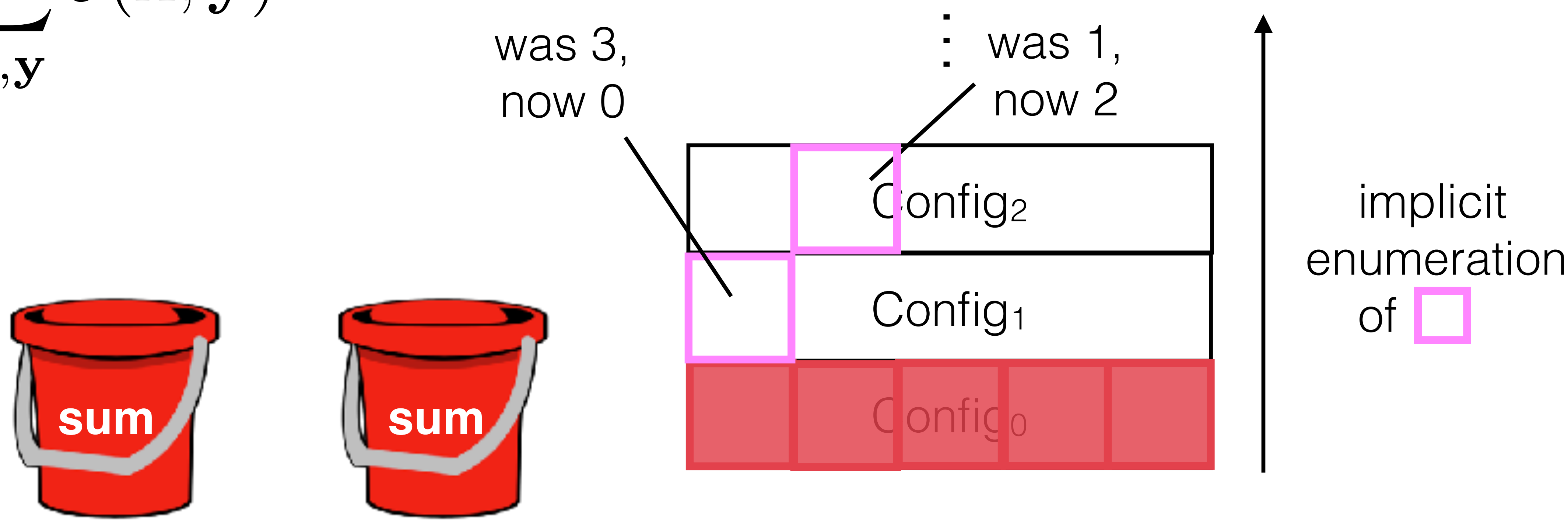

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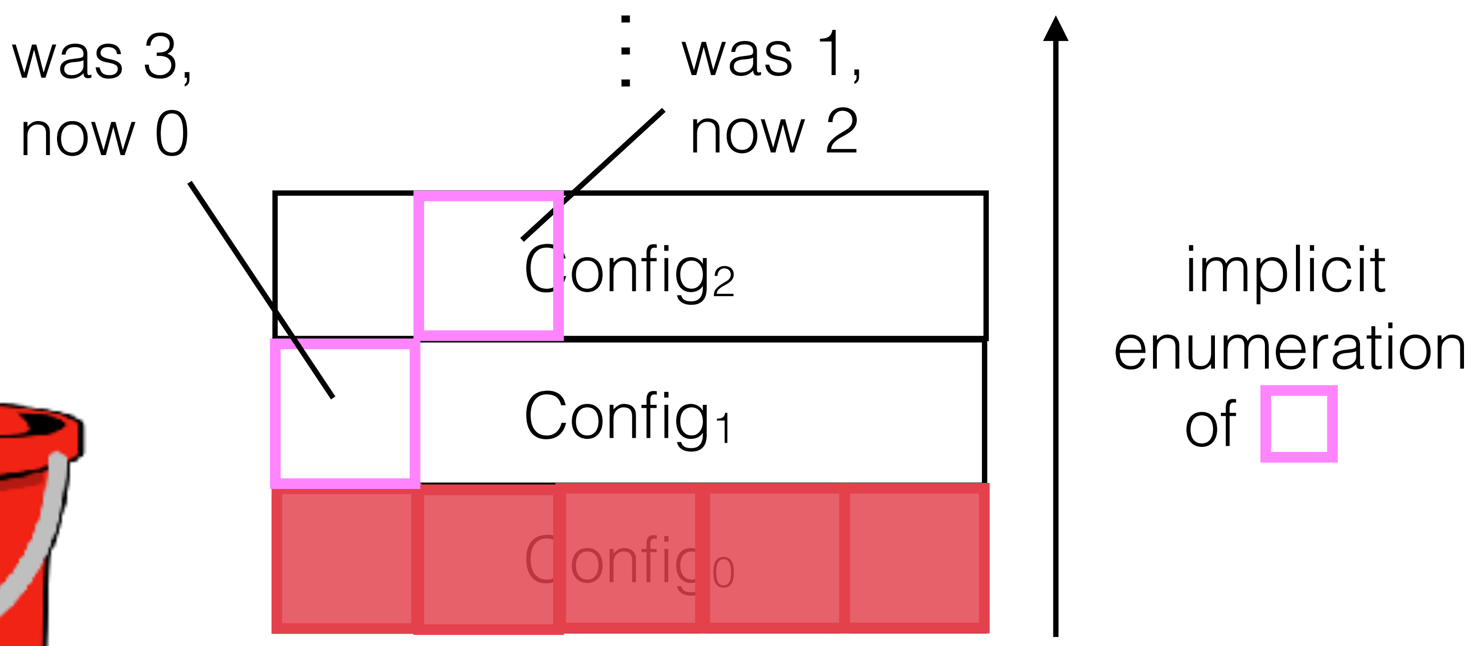

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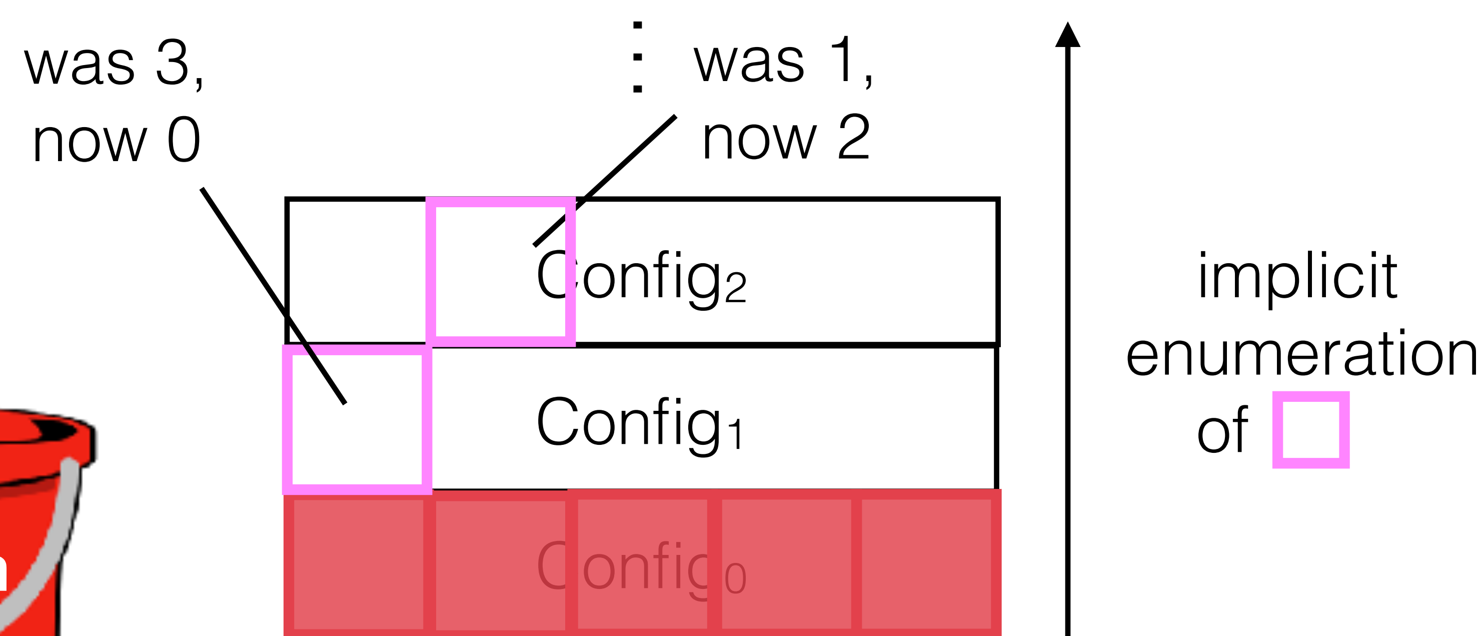

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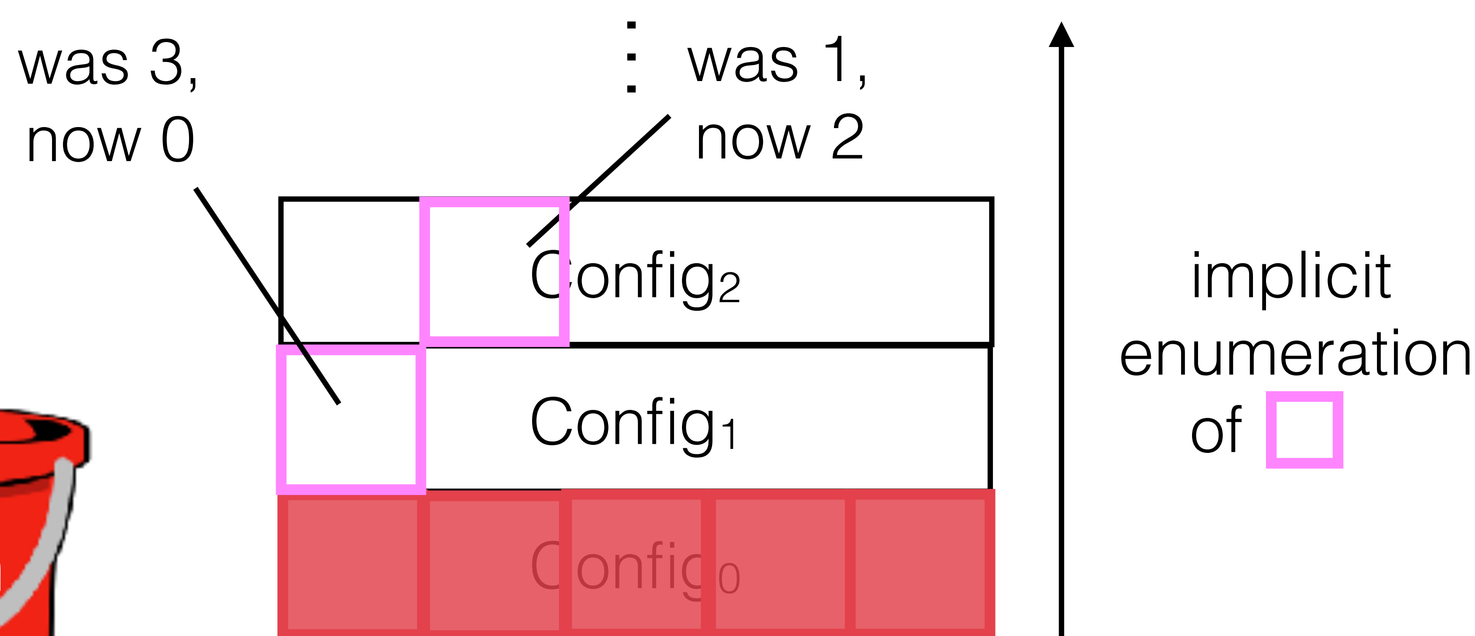

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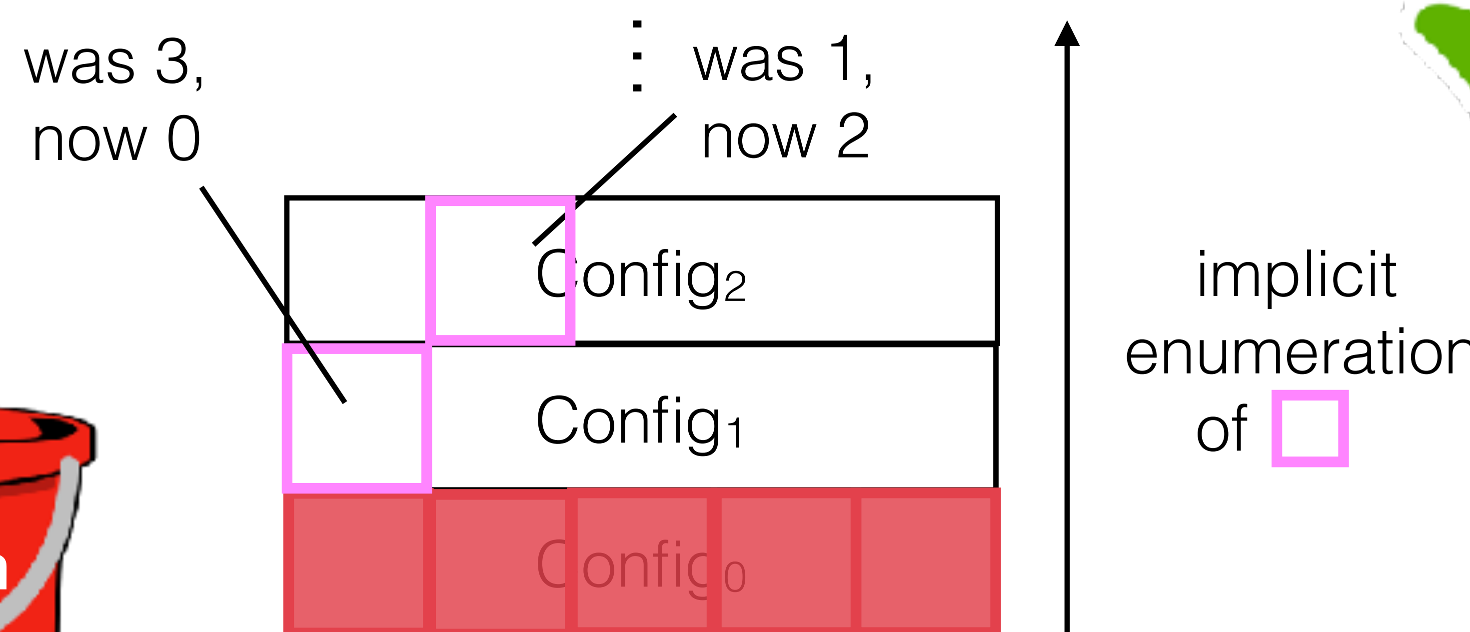

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  - Space stays $S + \mathsf{poly}(\kappa)$, not $S \cdot \mathsf{poly}(\kappa)$

Summary

	Assumptions	Prover Time	Prover Space
No-Signaling PCPs [KRR, ...]	PIR	$\geq T^3 S^3$	$\geq T^3 S^3$
SNARKs [BC,BCCT, ...]	Non-Falsifiable	$T \cdot \text{poly}(\kappa)$	$S \cdot \text{poly}(\kappa)$
Succinct Garbling [GHRW, KLW, ...]	Obfuscation/ multilinear maps	$T \cdot \text{poly}(\kappa)$	$S \cdot \text{poly}(\kappa)$
[this work]	"Slightly" Homomorphic Encryption	$T \cdot \text{poly}(\kappa)$	$S + \text{poly}(\kappa)$

Open Questions

- How does this compare in practice? What are the remaining bottlenecks?
- Can PCP query complexity be reduced?
- Is there an FHE scheme which is extra efficient for our prover?
    - Efficiently evaluate low-degree arithmetic circuits (large fields)

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low "asymmetric" degree (GSW) even better