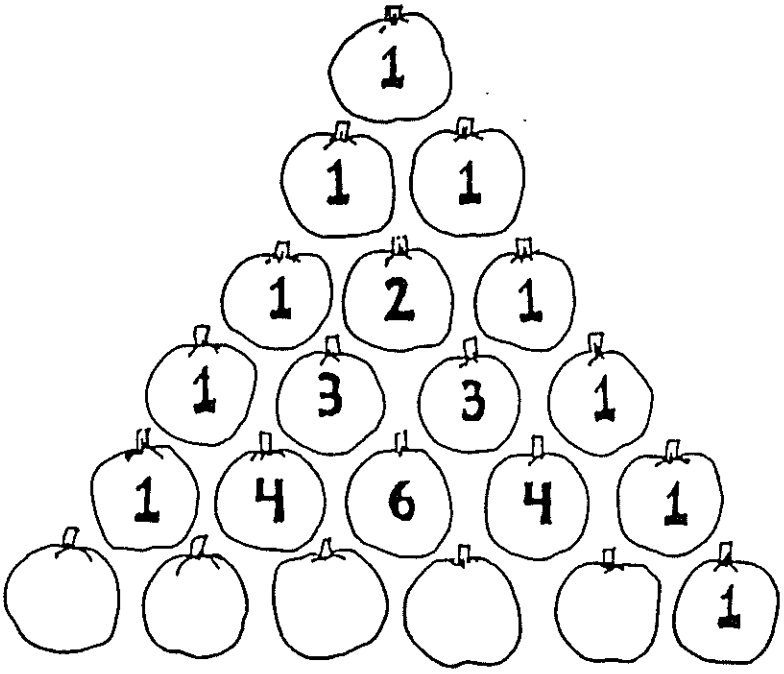
*Exploring Discrete Mathematics in the Classroom*

# *Powerful Patterns...Nifty Numbers K-4*

*A Workshop for Teachers About Patterns in Numbers*

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*in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Linda Bello, Carol Jacobs, Ann Lawrence, and Karen Wheeler*

# Powerful Patterns ... Nifty Numbers K-4

## Workshop Overview

### Summary

Participants will engage in a series of activities involving auditory, visual, and numerical patterns. Finding patterns in numbers helps students see the relationships between and among numbers. Practice with pattern recognition helps students develop the ability to generalize and therefore fosters mathematical power. Many activities involving exploring patterns are essential in K-4 math classrooms to help establish a framework for independent discovery and learning. This workshop provides a sampling of such activities, including triangular numbers, Fibonacci Sequence, handshake problem, and Pascal's triangle.

### Workshop Outline

I. Objectives

Participants will:

* discover a variety of patterns through exploratory activities,
* be exposed to a multitude of ways to teach patterns, and
* be provided with materials to implement the activities in their classrooms.

| | | |
|---|---|---|
| II. | Activity #1 — Introduction to Patterns | 15 Minutes |
| | A. Auditory Patterns | |
| | B. Visual Patterns | |
| | C. Symbolic Patterns (Worksheet can be done while waiting for workshop to begin) | |
| III. | Activity #2 — Exploring Triangular Numbers Using the Handshake Problem | 20 Minutes |
| IV. | Activity #3 — Pascal's Triangle | 25 Minutes |
| V | Activity #4 — Exploring the Fibonacci Sequence | 20 Minutes |
| VI. | Extensions (optional) | |
| | A. Coloring Pascal's Triangle (geometric/ visual patterns) | (20 Minutes) |
| | B. Possible Paths — Paths on a Grid | (20 Minutes) |
| VII. | Workshop Summary | 05 Minutes |
| VIII. | Workshop Evaluation | 05 Minutes |
| | Total Workshop Time: | 90 Minutes |

## Workshop Environment

I. Room Arrangement

Ideally, participants should be seated at round tables in groups of four to six persons; round tables help facilitate collaboration and discussion among group members. Each participant should have a reasonable view of the presenter and the screen.

II. Equipment

Overhead projector and screen

III. Workshop Materials List

A. Activity #1 — Introduction to Patterns

Transparency markers

B. Activity #2 — Exploring Triangular Numbers Using the Handshake Problem

Transparency markers
Twenty-one pennies

C. Activity #3 — Pascal's Triangle

Blank transparencies and transparency markers

D. Activity #4 — Exploring the Fibonacci Sequence

Post-It Notes: Largest size available so participants in the back of the room can see what number is written on each sheet.
Marking Pen: For writing numbers on post-it notes
Blank transparencies
Manipulatives of the animals in the story.

E. Extension Activity — Paths and Coloring in Pascal's Triangle

Blank transparencies and transparency markers
Colored pencils or markers for participants to use for coloring handout

# Instructor's Notes

## Introduction

Show the "Title Transparency" for this workshop and indicate that this workshop was developed by K-8 teachers who participated in the Rutgers University Leadership Program in Discrete Mathematics. Introduce yourself and mention that you spent three weeks over two summers learning discrete mathematics at the Leadership Program, and have been using materials from the program in your classroom.

Show the "Discrete Mathematics Transparency" and review the information there — discussing five major themes of discrete mathematics. Indicate that the workshops offered in the "Workshops in Your District" program reflect four of these major themes. The theme that we will be focusing on in this workshop is that of "Patterns"; the title of this workshop, as noted on the "Title Transparency" is "Powerful Patterns ... Nifty Numbers".

Although you may have been given this information beforehand, find out the grade levels of the participants in your workshop (say, for K, 1-2, 3-4, 5-6, and 7-8) and, if the distribution is not what you expected, be sure to modify your workshop appropriately. Also, please tell the participants that all the information and all the activities in this workshop will be included in a packet of materials that they will receive after the program.

## Activity #1 — Introduction to Patterns
(Allocated time = 15 minutes)

A. Auditory Pattern: Get the group's attention by performing an auditory pattern, e.g., Clap, Clap, Snap, Snap, Clap, Clap, Snap, Snap, ... until the group repeats your behavior.

*K-4 teachers are usually quick to join in on kinesthetic activities. If needed, prompt with some encouraging remarks, such as, "Join me." After a few seconds, stop and ask participants to verbally describe the pattern. Lead them to observe that auditory patterns can also be represented as visual patterns. For example, A, A, B, B, A, A, B, B, ... can be used to represent the above auditory pattern.*

B. Visual Pattern: Ask participants to take the auditory pattern and generate some corresponding visual pattern that makes sense to them. A variety of responses are appropriate here and participants should be encouraged to generate as many different visual patterns as possible.

*Participants use visual characteristics of other workshop participants (e.g., gender, height,*

*hair color, clothing, etc.) to line up people at the front of the room following the same pattern, A, A, B, B, A, A, B, B, ... .*

**C. Symbolic Patterns: Using TSP #1, instruct participants to find the next two terms in each pattern. Allow them to work for a few minutes with those around them to discuss possible patterns; encourage them to identify a variety of appropriate solutions. Solicit suggestions for each pattern. Emphasize that there are several ways to continue each sequence.**

*The first sequence might be interpreted as having its first term containing 1 curve, second term containing 2 curves, third term containing 4 curves and the next element in the sequence should then contain 8 curves, followed by 16 curves. One can imagine a variety of ways to create those figures. A second way to view this sequence may be by focusing on lines of symmetry. That is to say, the first element can be viewed as the numeral "one" and its reflection over a vertical line of symmetry. Continuing in this way, the second element may be viewed as the numeral "two" and its mirror image over a vertical line of symmetry, followed by three and its mirror image. In this case the next two elements would be the numeral "four" and its mirror image over a vertical line, followed by "five" and its mirror image. Of course, other suggestions may be offered as well. The second sequence might be interpreted as simply words in alphabetical order. Others may view this sequence as a 3 letter word, a 4 letter word, a 5 letter word, to be followed by a 6 letter word, and a 7 letter word, etc., OR as girl's name, boy's name, girl's name, boy's name, girl's name , etc., OR as girl's name with letter A, boy's name with letter B, girl's name with letter C, to be followed by boy's name with letter D, and girl's name with letter E, etc. The third sequence might be interpreted as a number with a three-letter name, a number with a four-letter name, a number with a five-letter name, followed by a number with a six-letter name (e.g., ELEVEN), and a number with a seven-letter name (e.g., FIFTEEN), etc. Another pattern might be found by using the rule "adding three to the previous element followed by subtracting one from the previous element" which provides the following sequence: One, Four (1+3), Three (4-1) followed by Six (3+3), Five (6-1), Eight (5+3), Seven (8-1), etc.*

***Option:** At the close of an activity (either here or later in the workshop) you may want to emphasize the importance of communication, especially writing, in the mathematics curriculum. Ask the participants to write what they think students might learn from the activity, and ask a few participants to share what they wrote.*

**Activity #2 — Exploring Triangular Numbers using the Handshake Problem**
**(Allocated time = 20 minutes)**

**Introduce this activity by saying, "You have done such a wonderful job on these patterns. Give a handshake to each person in your group to congratulate them on a job well done. Be sure to shake each person's hand exactly once. Count the total number of**

handshakes needed in your group. You may need to keep track of the handshakes to make sure that every person in fact shook hands with every other person exactly once."

Point out that the question of "How many handshakes?" is a traditional problem of discrete mathematics which is known as "The Handshake Problem". Show TSP #2 and enter into the chart some of the numbers that were generated by the groups at the tables. If there is more than one table with five participants, for example, check to see whether they all got the same answer; if they didn't, record the different answers in the T-chart, and note that we'll have to determine in a while which of the answers is correct. Hand out HO #1 and encourage the participants to work in their groups to fill in the chart with the missing values. When each group finishes, they should check their answers with those of another group to make sure that they agree on an answer. Then they should be asked to discover and explain the pattern that emerges from The Handshake Problem.

*The completed t-chart for 8 people appears below.*

| # of People | # of Handshakes |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 3 | 3 |
| 4 | 6 |
| 5 | 10 |
| 6 | 15 |
| 7 | 21 |
| 8 | 28 |

*What you hope they will discover is that to get from each number to the next you have to add first 1, then 2, then 3, then 4, then 5, then 6, etc. Or they might discover that each number in this sequence is a sum of consecutive counting numbers — 1, 1+2, 1+2+3, 1+2+3+4, 1+2+3+4+5, etc. What you hope they will explain is that one way in which all of the handshakes can be accomplished is that the second person shakes hands with one person, the third person shakes hands with two people, the fourth person shakes hands with three people, the fifth person shakes hands with four people, etc.*

Line up 7 volunteers in the front of the room, and have a demonstration that the 21 handshakes involving 7 people is 1+2+3+4+5+6. (Do this even if someone comes up with this explanation.) On the overhead projector, place 21 pennies in a triangular formation and ask how many pennies there are. They should recognize that the total is 1+2+3+4+5+6, which is 21. Point out that all of the numbers in the T-Chart can be represented by triangles made out of pennies. For this reason they are called "Triangular Numbers".

**Activity #3 — Pascal's Triangle**
**(Allocated time = 25 minutes)**

**A. Introduce Pascal's Triangle, using TSP #3. Ask participants, "What patterns do you see?" Allow them to verbalize observations like "I see all 1s along two sides of the triangle" and/or "I see any term generated by the sum of the two numbers directly above that term." Allow participants to identify any other patterns, but don't address them in detail at this point. Ask participants to provide values for the last row on TSP #3.**

*Even though some of the K-4 teachers may have seen Pascal's Triangle before, most will probably not be familiar with it and may not realize at first how the terms are generated. You may need to lead them to discover how these terms are generated by asking questions such as, "Focus on the number 6 in the row 1,4,6,4,1. Are there any numbers above the 6 which might generate that value?" In addition, you will need to explain that this row (1,4,6,4,1) is the "fourth" row of Pascal's Triangle since its first entry (except for the 1 on the left) is 4. (Working backwards, you should note that the top 1, which is in a row by itself, is called the $0^{th}$ row.)*

**B. Have participants work in groups to complete HO #2 = TSP #4. Use TSP #5 as an overlay on TSP #4 to display the correct values once participants have found them. Show TSP #6 and say, "This transparency shows rows 0 to 16 of Pascal's Triangle. Notice that it contains large numbers fairly quickly."**

*It is important to mention to your workshop participants that students at grades K-4 need to be told that this pattern can be generated for as many rows as they wish. Students at these grade levels will tend to think that Pascal's Triangle ends wherever the visual triangle that you show them is completed unless you specifically mention that Pascal's Triangle goes on forever.*

**Now ask the participants to work in groups and find the counting numbers (starting with 1) and the triangular numbers as patterns in Pascal's Triangle. These patterns may have been explored earlier in the workshop, but not located in Pascal's Triangle. Participants should find the following solutions, which you should demonstrate on TSP #6:**

1. the Counting Numbers

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

2. the Triangular Numbers

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

**C. Have participants explore another pattern found in Pascal's Triangle. Ask participants to add the values in each row and look for a pattern in the sums. Use a blank transparency to organize their findings as shown below:**

| Row Number | Sum of Each Row |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |

**Ask participants, "What pattern do you see in the Sums column?" After your group has identified the pattern as "the powers of two," add the third column (as shown below) to your transparency.**

| Row Number | Sum of Each Row | Powers of 2 |
|---|---|---|
| 0 | 1 | $2^0$ |
| 1 | 2 | $2^1$ |
| 2 | 4 | $2^2$ |
| 3 | 8 | $2^3$ |
| 4 | 16 | $2^4$ |
| 5 | 32 | $2^5$ |

*Some K-4 teachers may not remember exponents if they have not worked with them for some time. If you present this in a workshop and notice blank stares on the participants faces, you may need to "refresh" their memory that $2^4$ means 2 times 2 times 2 times 2 (i.e., the base number — two, in this example — times itself the exponent number of times — four, in this case). Some participants may wonder about the $0^{th}$ row and why $2^0$ is 1. You might simply say that the convention that any number to the 0 power is 1 makes many formulas work out correctly; however, you shouldn't dwell on this issue. Of course, if participants want to learn more about this, you should discuss it with them afterwards.*

**Ask participants, "What is the sum of the 10th row of Pascal's Triangle?" They should realize that it is $2^{10}$, which is a little over 1000 (it is exactly 1024). Some participants might observe that the sum of the $n^{th}$ row of Pascal's Triangle is $2^n$, but don't dwell on this, since K-4 teachers are often intimidated by the algebra.**

**D. Show TSP #7. Ask participants, " How many ways can you find to spell CATS in this array? Each letter must be on the row just below the previous letter." Pass out HO #3 = TSP #7. Give participants time to trace paths and count. Solicit solutions. If possible, have a volunteer participant relate the solution to the previous activity about paths on a grid and, therefore, to Pascal's Triangle.**

*The number of paths to any one position (letter) can be determined by adding the total number of paths to the two above it, as shown in the diagram on the next page:*

C ①

A ① A ①

T ① T ② T ①

S ① S ③ S ③ S ①

*The numerals beside each letter is the total number of paths to that letter. Thus, when you sum the total number of paths in the "S" row, there are 8 ways to trace CATS in the array.*

**Activity #4 — Exploring the Fibonacci Sequence**
**(Allocated time = 20 minutes)**

**A. Generating the Fibonacci Sequence: Pass the Pad. In this activity participants generate the Fibonacci sequence on post-it notes. Solicit ten volunteers to stand in a row across the front of the room. Give the first participant a large post-it pad and a magic marker. Ask the participant to write the numeral "1" on the top page of the pad. Ask that person to tear off the page, stick it on him/herself, and pass the pad and marker to the next participant. The second participant should do the same. Each subsequent participant should, in turn, write the sum of the two previous participants' numbers on the top page of the pad, put it on him/herself, and then pass the pad and marker to the next participant. Explain that the participants have generated the Fibonacci Sequence of numbers. The first person has the first Fibonacci number, the second person has the second Fibonacci number, the third person has the third Fibonacci number, etc.**

*The third participant writes the sum (2) of the two previous participants' numbers (1+1). The fourth participant writes the sum (3) of the two previous participants' numbers (2+1). This pattern continues: 1, 1, 2, 3, 5, 8, ... For each successive element, have the audience predict its value.*

**B. Connecting the Fibonacci Sequence to Children's Literature. The following activity is part of the "All Aboard" activity in the workshop entitled *Connecting Children's Literature With Discrete Mathematics Topics — Grades K-4.***

**Introduce this activity by placing TSP #8 on the overhead projector and reading the following summary: ""Hey! Get Off Our Train" by John Burningham is a counting book that acquaints readers with the topic of endangered species. In this book, a boy and his pajama-**

**case dog climb aboard the toy train and embark on a glorious trip around the world. In their journey they meet representatives of endangered species who wish to join them."**

**Say:- "You can use this story to introduce the Fibonacci Sequence of numbers in the following way. Suppose that the animals can board the train either one at a time or two at a time. In how many different ways can the animals board the train by ones and twos?" Let us suppose that there are just two animals — the elephant and the seal -- and that they have to board in that order. (It would be helpful to have "elephant" and "seal" manipulatives at this point.) The elephant could go first and then the seal, or the elephant and seal could board together — so there are two different ways in which they could board. Let's try it with three animals — the elephant, the seal, and the crane. Ask the participants to figure out in how many ways they could board the train if they are allowed to board two at a time, but must board in that order. After a few minutes, review all of the possibilities on the board (or on the overhead projector). Either they could go on one at a time, or the elephant and seal could go on together (and then the crane), or the elephant could board alone followed by the seal and crane together — so that there are three ways altogether.**

**Pass out HO #4 (which is also TSP #9) and ask participants to work in pairs to fill in the chart. Remember: the animals may board one or two at the time and they always board in the same order: elephant, seal, crane, tiger, then polar bear. When nearly all have finished, let one or more pairs share their findings on a blank transparency. Then ask, "Now predict the total number of ways that ten animals can board the train by ones or twos." Using the Fibonacci sequence, they should be able to predict 55 ways. Summarize the activity and answer any questions.**

*As you circulate, be sure to emphasize that the animals must board the train in the order specified. Also, some pairs may need help in getting started. Review with them that the elephant, seal, and crane can enter in three ways: one(elephant), one(seal), then one(crane); two(elephant and seal), then one(crane); or one(elephant), then two(seal and crane) or numerically as 1-1-1, 2-1, or 1-2*

| *# of Animals* | *Ways to Board* | *Total # of Ways* |
|---|---|---|
| *1* | 1 | 1 |
| 2 | 1-1, 2 | 2 |
| 3 | 1-1-1, 2-1, 1-2 | 3 |
| 4 | 1-1-1-1, 1-2-1, 2-1-1, 1-1-2, 2-2 | 5 |
| 5 | 1-1-1-1-1, 2-1-1-1, 1-2-1-1, 1-1-2-1, 1-1-1-2, 1-2-2, 2-1-2, 2-2-1 | 8 |

*Many K-4 classroom teachers often use literature as the vehicle to instruct other curriculum areas, including mathematics. Other books appropriate for this topic include the following:*

Anno, Mitsumasa. *Anno's Magic Seeds*. NY: Philomel Books, 1982.

Barry, *David. The Rajah's Rice.* NY: WH Freeman, 1994.
Birch, David. *The King's Chessboard.* NY: Dial Books for Young Readers, 1988.
Chwast, Seymor. *The 12 Circus Rings.* NY: Harcourt Brace, 1993.
Crews, David. *Ten Black Dots.* NY: Greenwillow Books, 1986.
Greene, Carol. *Thirteen Days of Halloween.* NY: Children's Books, 1988.
Grossman, Bill. *My Little Sister Ate One Hare.*NY: Crown, 1996.
Grossman, Virginia and Long, Sylvia. *Ten Little Rabbits.* San Francisco: Chronicle Books, 1991.
Merriam, Eve. *12 Ways to Get 11.* NY: The Trumpet Club, Inc., 1993.
Pittman, Helena Clare. *A Grain of Rice.* NY: Bantam Skylark, 1986.
Ross, Pat. *M&Ms.* NY: Viking Press, 1985.
Scleszka, Jon and Smith, Lane. *Math Curse.* NY: Viking, 1995.

**Extension Activity 1 (optional) — Coloring Pascal's Triangle**
**(Allocated time = 20 minutes)**

**Using TSP #10 and HO #5, help participants find various ways to create geometric/visual patterns in Pascal's Triangle by using mathematical rules. Say: "Some of the patterns in Pascal's Triangle can be seen best as visual or geometric patterns. One way you can discover such a pattern is to choose any whole number from 2 to 10. Color in all spaces corresponding to values in Pascal's Triangle which are multiples of the number you choose. Using one color will make the pattern stand out more." You may choose to demonstrate this method by using a blank transparency as an overlay for TSP #10 and color all multiples of two in the first few rows of Pascal's Triangle red. Ask each table to use HO #5 and to color any set of multiples the same color. While they're working, display TSP #6 again.**

*It may be helpful to show participants student work while they are coloring their triangles. Point out that students can often discover visual/geometric patterns after completing several rows.*

**Show TSP #11 and explain to participants the patterns generated for multiples of 2 through 10.**

**Extension Activity 2 (optional) — Paths in a Grid**
**(Allocated time = 20 minutes)**

**In this activity, participants explore the relationship between the total number of paths on a grid and Pascal's Triangle. Show TSP #12 (with a 2x3 grid) on the overhead projector and say: "There are several different paths to get from A to B. On this walk, you can only move down (south) or right (east) on each move. How many different ways are there of doing this?" After they have a chance to digest the problem, put up TSP #13 and announce: "A powerful problem solving technique that mathematicians often use is "Solving a simpler**

problem". Here we have two simpler problems. How many ways are there to get from A to B in the first diagram (with a 1x1 grid)? That is, starting at A in the top diagram, in how many ways can you move one block right and one block down?" Wait for a participant to indicate, "Two ways: either south east OR east south." "Now let's try the next problem (with a 1x2 grid)." Work through with them the listing of all 3 solutions and also the method of labeling each vertex with the number of paths to that vertex. Then hand out HO #6 and instruct the participants to work in groups to solve the problem. Allow time for participants to work in groups as you circulate among them. Encourage them to systematically list the paths they find. Volunteers can share their solutions using blank transparencies as overlays on TSP #12.

*You may find that some groups have difficulty with this activity because they will need to see the different possible paths. In this case, you may want to suggest that they list or draw out all ten possible paths on another sheet of paper. After participants have shared their solutions and methods for solving the problem, you will need to demonstrate the following method if it was not employed: Use numerals at each vertex to indicate the total number of paths to that vertex. Explain how and why the numbers are obtained, using a blank transparency as an overlay on TSP #12.*

Ask participants if they see any patterns in these numbers. Once they announce that it's Pascal's Triangle, turn TSP #14 sideways, and overlay TSP #12 so participants can see the visual connection. You will need to explain that the numbers outside the grid would be the numbers generated if the grid was expanded (these numbers are also the numbers which complete the second, third, and fourth rows of Pascal's Triangle.)

*Lead participants to observe that a rectangular grid of any size relates to Pascal's Triangle in a similar way.*

## Workshop Summary

Allow some time for participant questions, then explain that this workshop has only "touched the tip of the iceberg"! There are many other patterns found in Pascal's Triangle and many other interesting patterns in number sequences. For example, the Fibonacci Numbers are found many places in nature and a whole workshop can be devoted to that topic alone! Solving problems that involve patterns helps students develop problem-solving skills and therefore fosters mathematical power. This workshop provides a sampling of meaningful mathematical activities that can be used in K-4 classrooms.

Remind them that they will soon receive a packet of "Take-home Materials" (TKHM) which contains the materials used in the workshop and (in some cases) additional materials. Encourage participants to review these materials and use the activities in their classrooms. In

addition, they should be encouraged to develop other activities through which they can weave patterns throughout their curriculum.

## Workshop Evaluation

Pass out the "Participant Evaluation Forms". Encourage participants to add their constructive suggestions for future presentations of this workshop. Be sure to provide them with five minutes to complete the form.

Collect forms from participants and give them the "Take-home Materials". Draw their attention to the last page of the "Take-Home Materials", and encourage them to record on that page their impressions of the workshop and their thoughts about how they might use the workshop materials in their classrooms.

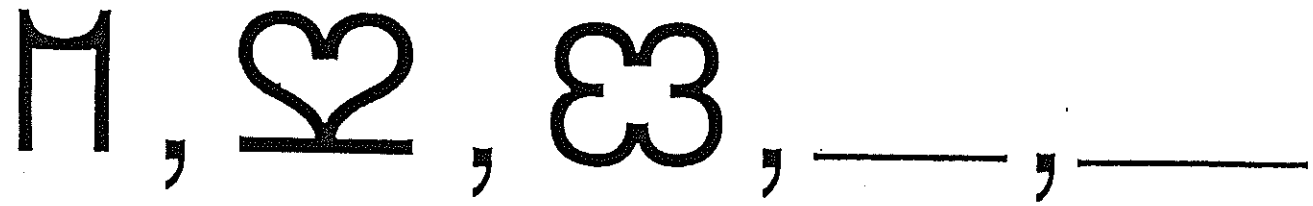
Immediately following the workshop (or at the latest that evening when you get home) please complete the "Presenter Report Form" and then the "Participant Summary Form".

Within three days of your workshop, please mail all "Participant Evaluation Forms", the Presenter Report Form", and the "Participant Summary Form" — together with an annotated copy of these Instructor's Notes and copies of any supplementary materials that you used — to the following address:

K-8 Workshop-Workshop Evaluation Information
P.O. Box 10867
New Brunswick, NJ 08906

# SYMBOLIC PATTERNS

**Find the next two terms in each pattern.**



**Amy, Bart, Carol, _____ , _____**

**One, Four, Three, _____ , _____**

# THE HANDSHAKE PROBLEM

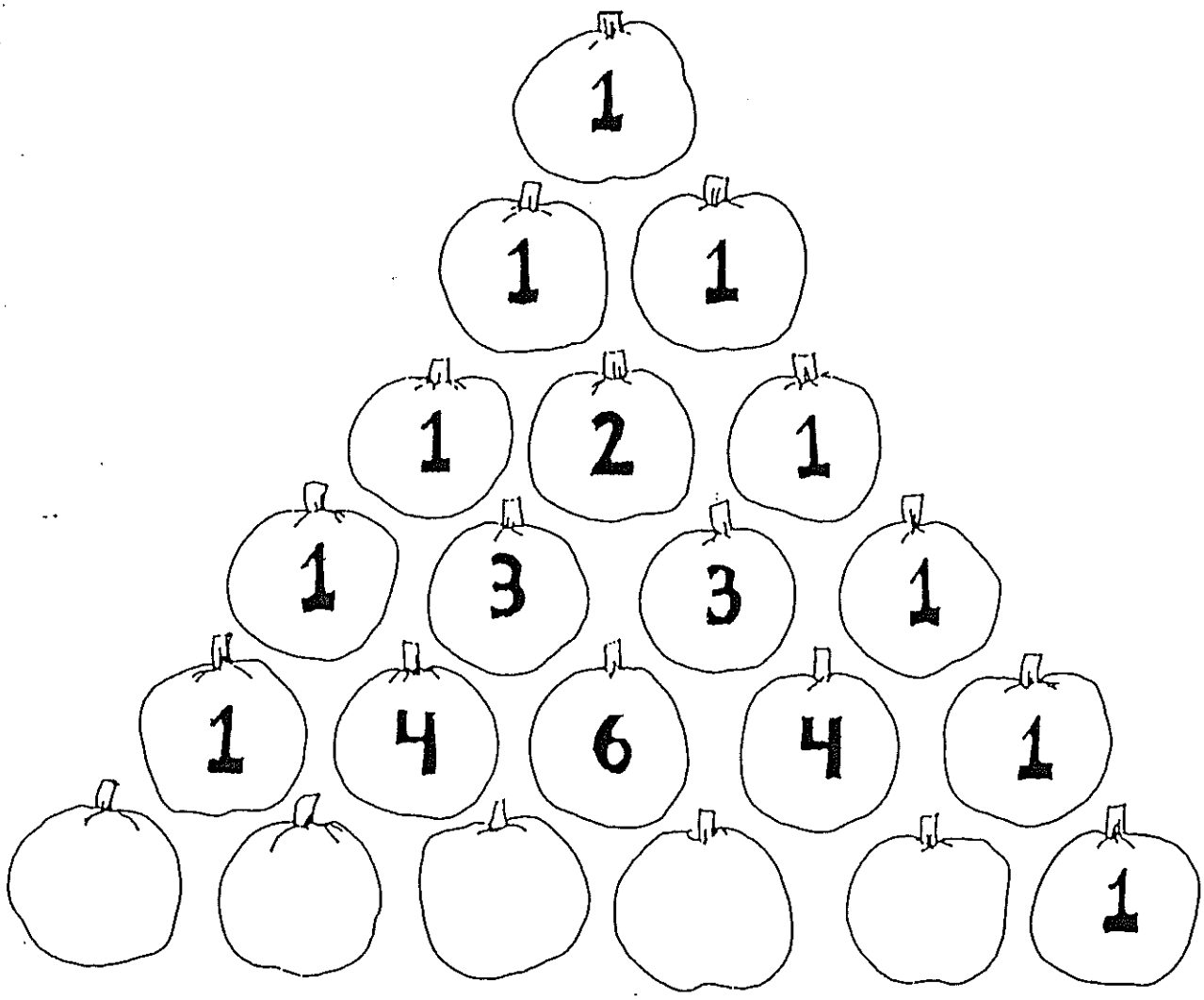
Suppose there are 8 people in your group and each person shook hands with every person in the group. How many handshakes would there be altogether? Solving simpler problems and making a T-Chart can help you solve the problem by finding a pattern.

If there was 1 person, there would be 0 handshakes. With 2 people, there would be 1 handshake. How many handshakes with 3 people? 4 people? 5 people? Fill in the chart below to find the pattern.

| Number of People | Total # of Handshakes |
|---|---|
| 1 | |
| 2 | ____ |
| 3 | ____ |
| 4 | ____ |
| 5 | ____ |
| 6 | ____ |
| 7 | ____ |
| 8 | ____ |
| | ____ |

# PASCAL'S PUMPKINS



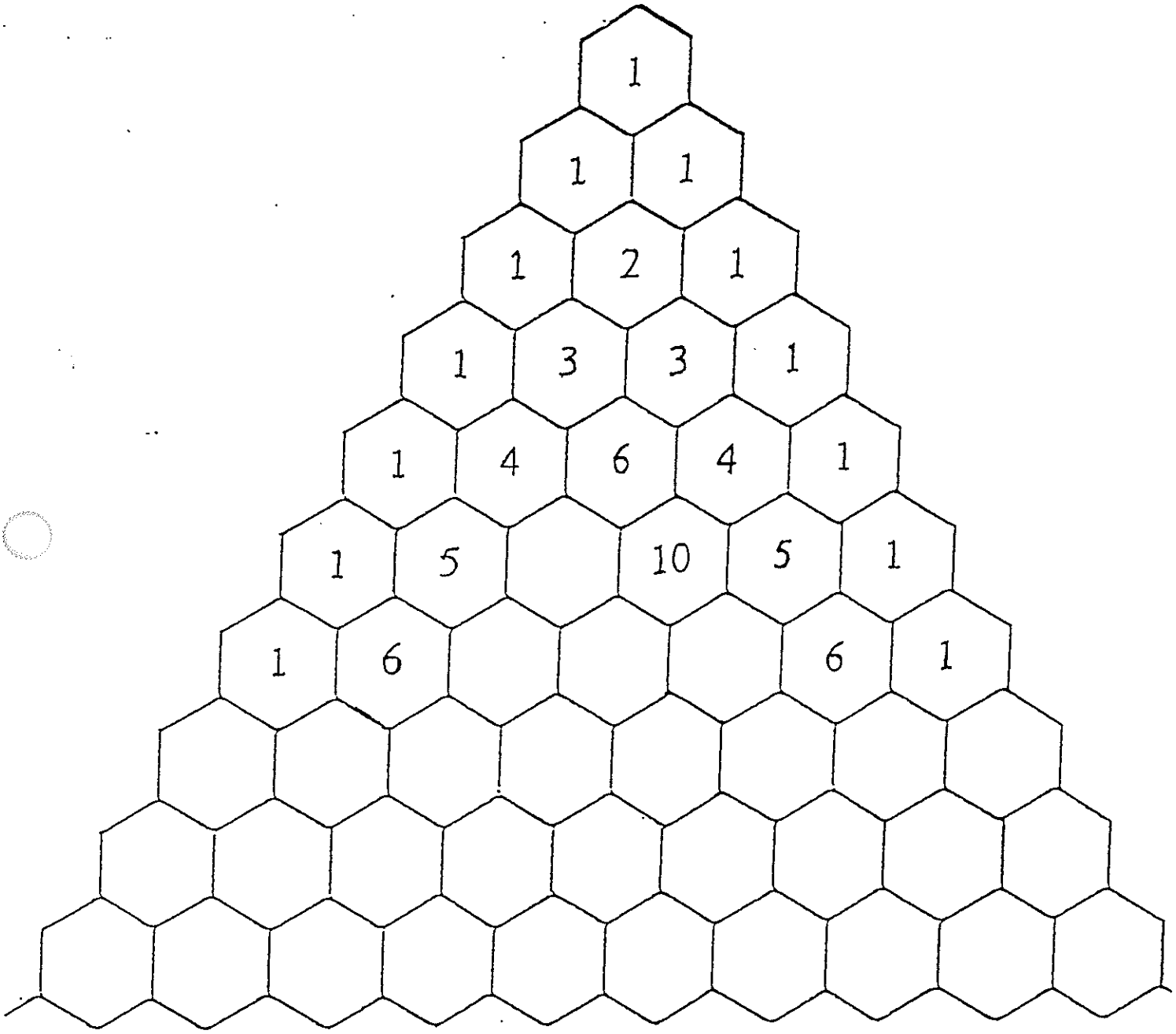

**What numbers belong on the bottom row of pumpkins?**

**Add the numbers in each row. What kind of pattern do you see?**

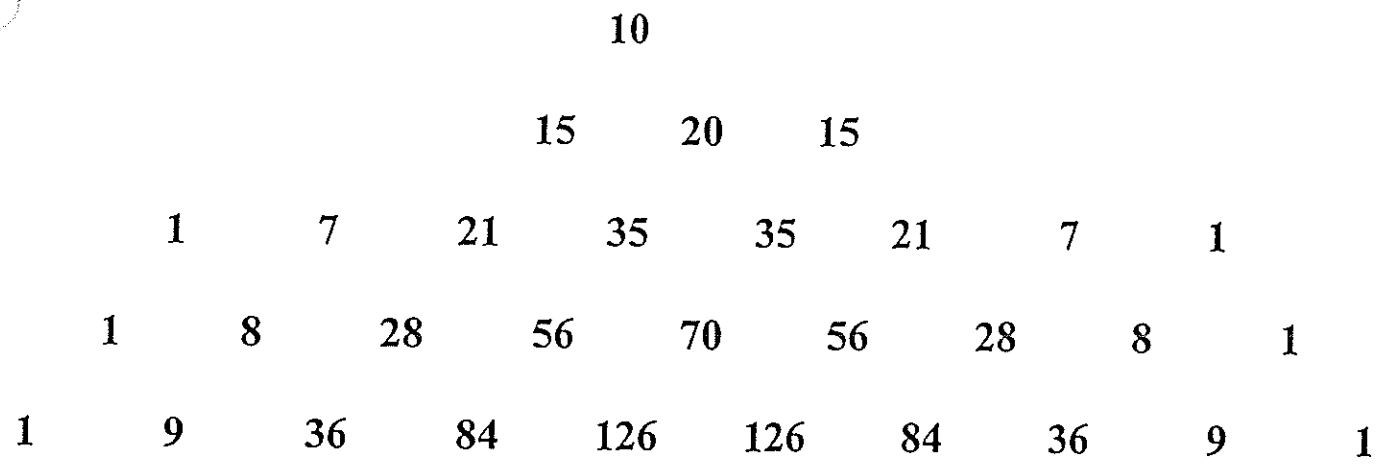
**What other patterns can you find in Pascal's Pumpkin Patch?**

# PASCAL'S TRIANGLE

**Complete the next two rows of Pascal's Triangle.**

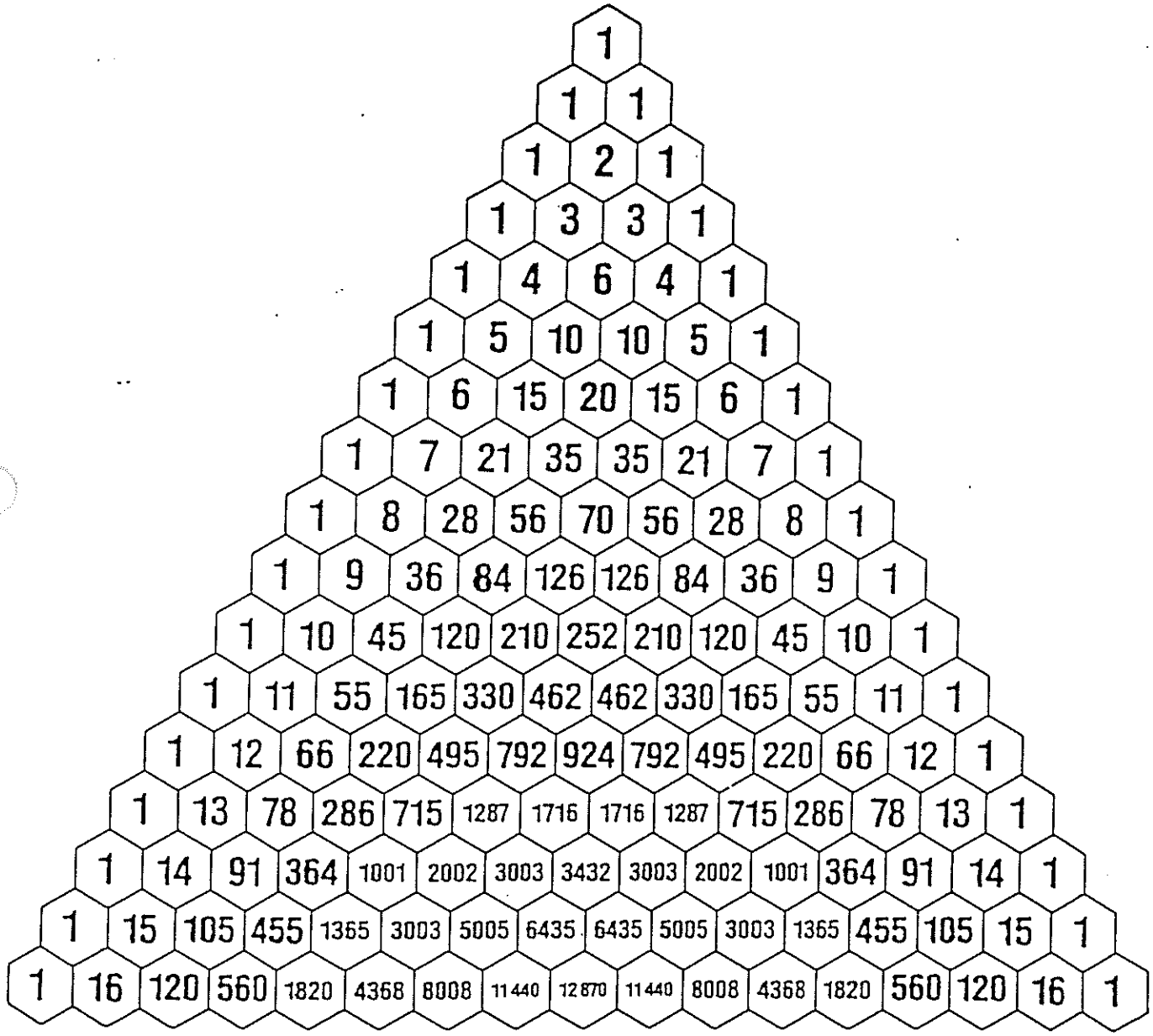




# PASCAL'S TRIANGLE



# CATS

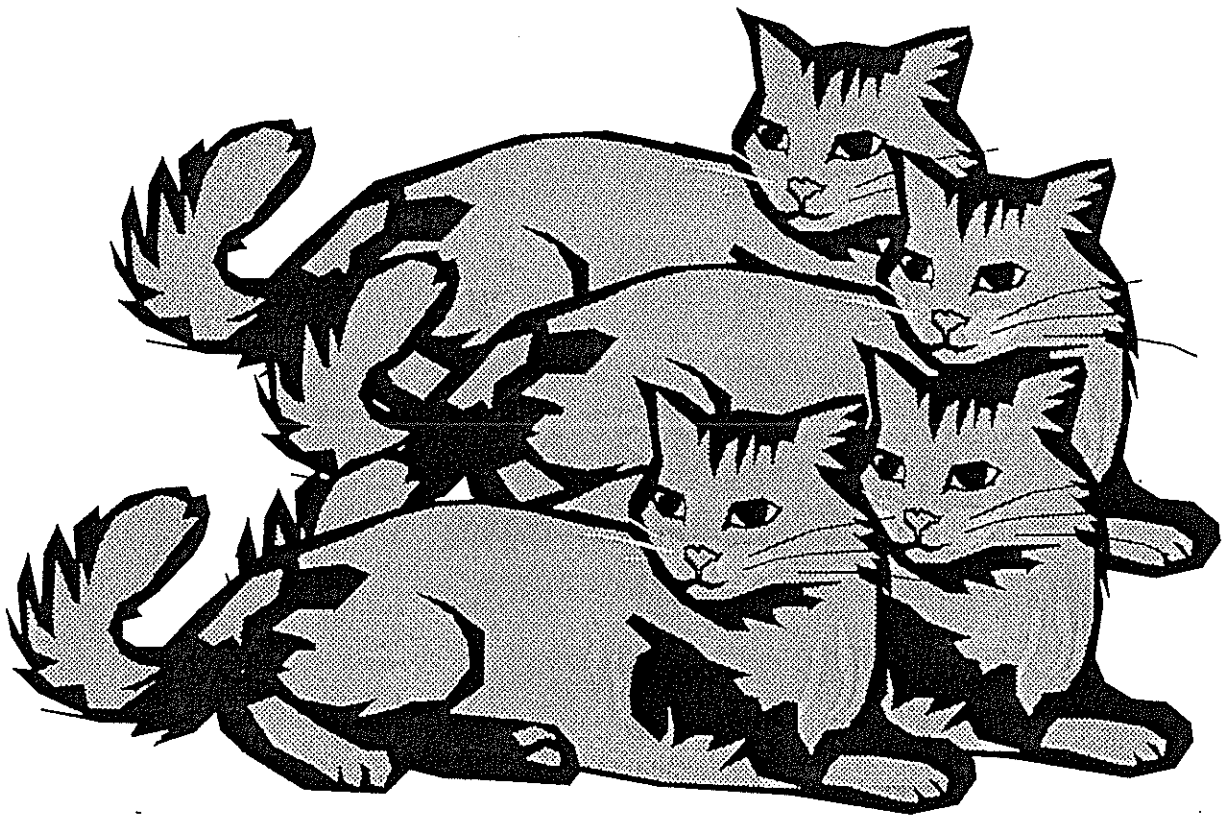
## How many ways are there to spell CATS?

C

A A

T T T

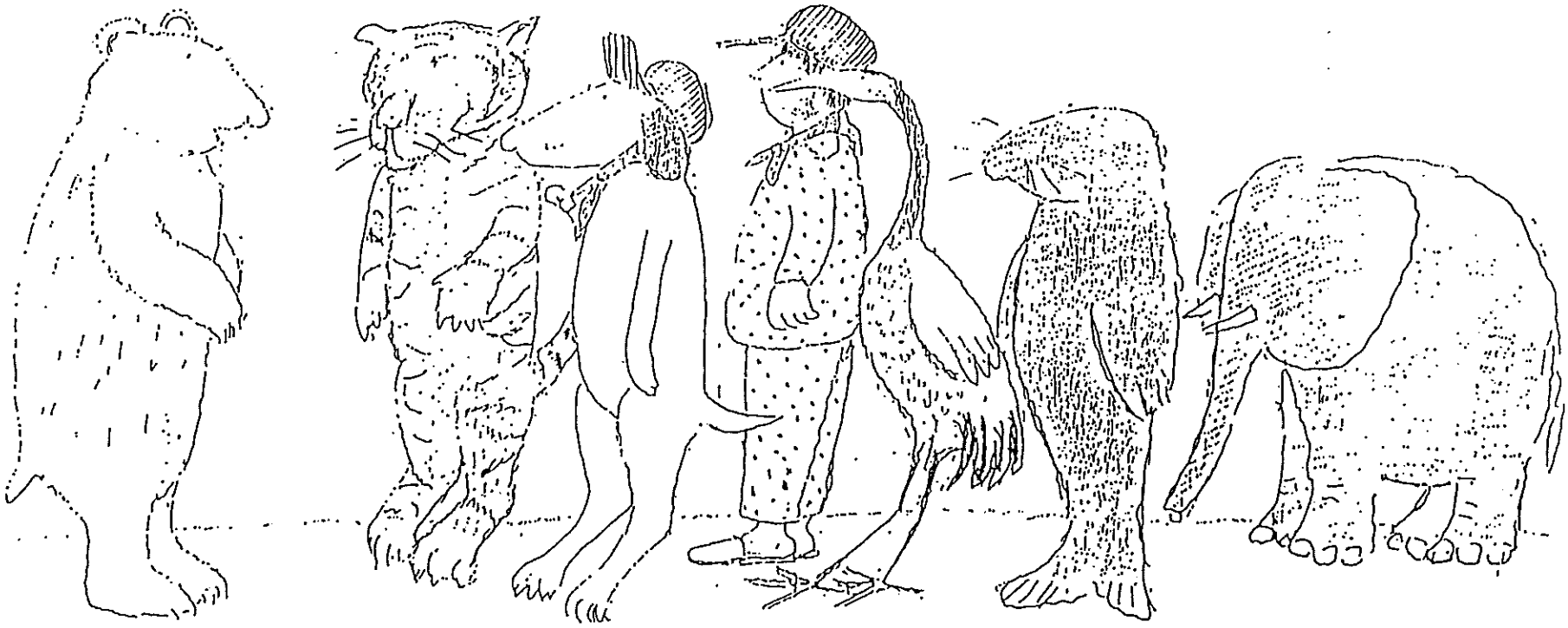
S S S S



# Hey! Get Off Our Train

## John Burningham



# ALL ABOARD

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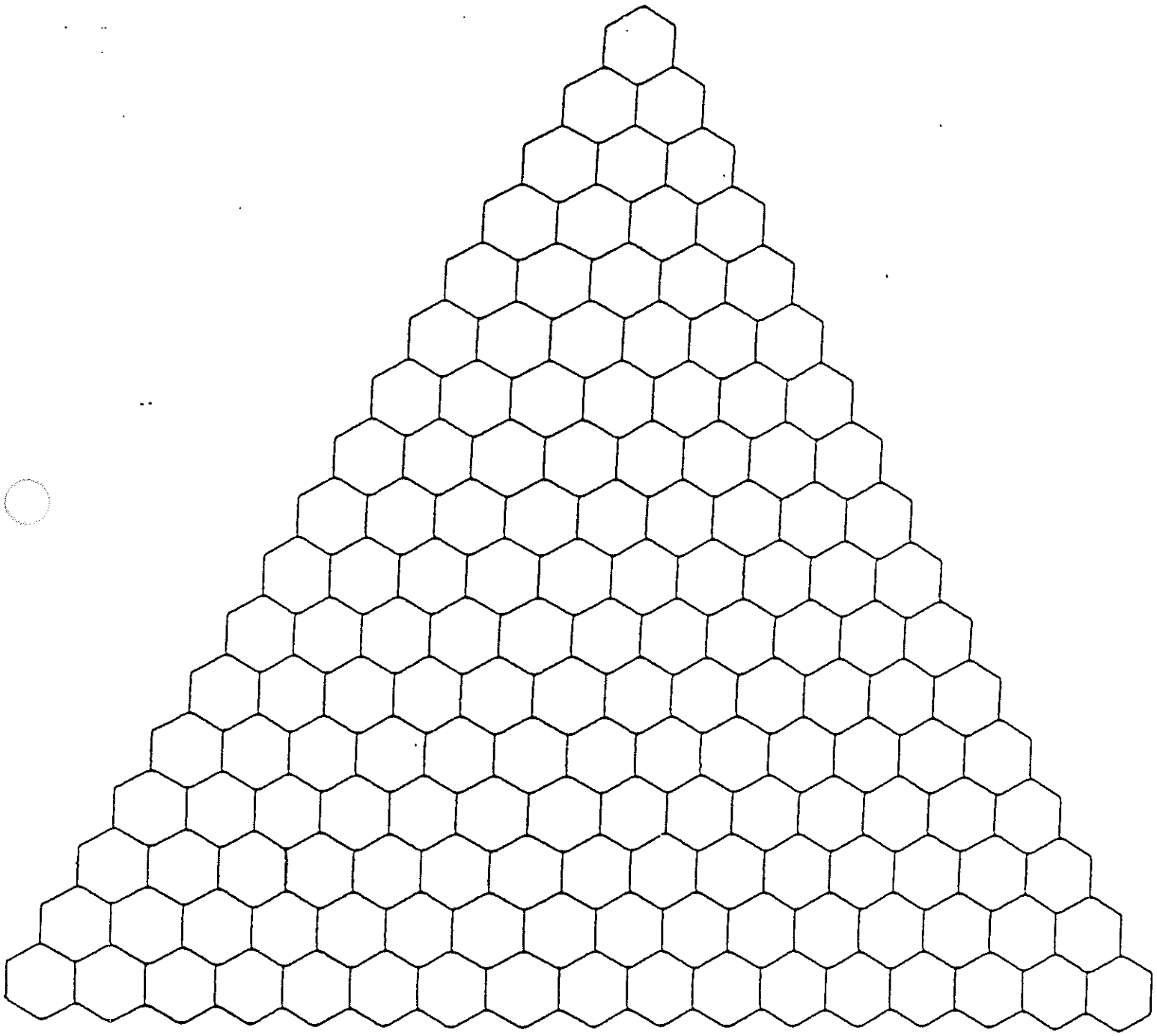
| Number of Animals | Ways to Board | Total # of Ways |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| | | |

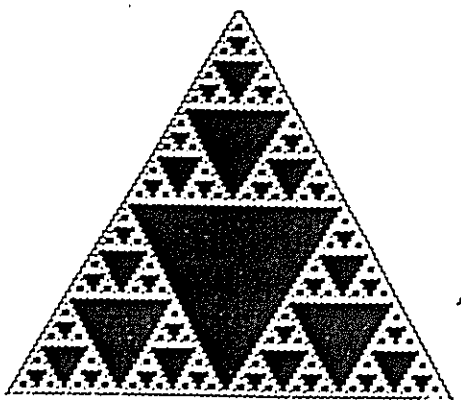
 

# COLORFUL PATTERNS IN PASCAL'S TRIANGLE

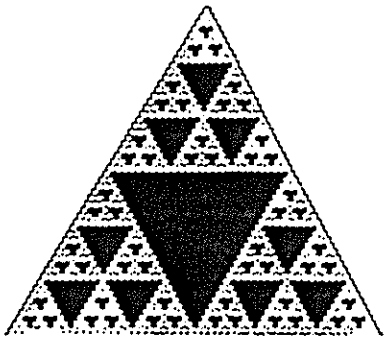


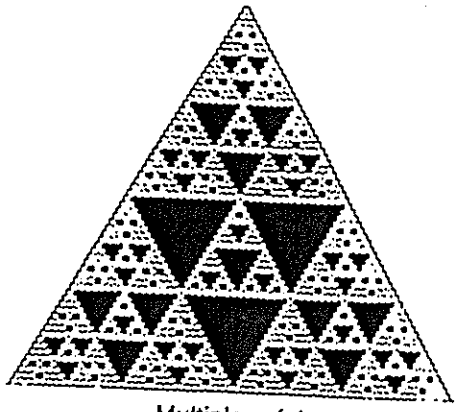
# VISUAL PATTERNS IN PASCAL'S TRIANGLE



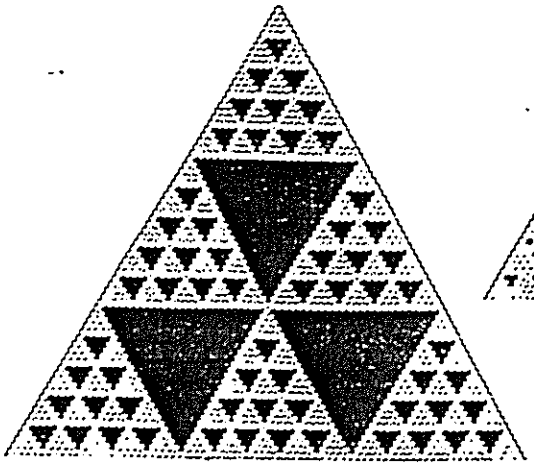
Multiples of 2



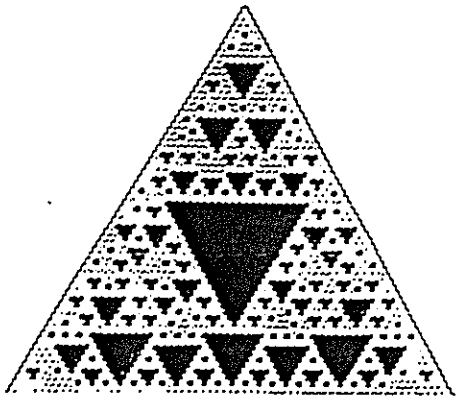
Multiples of 3



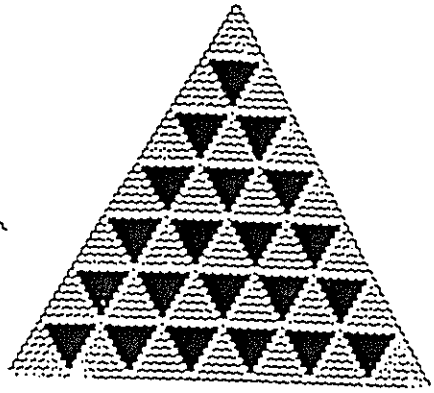
Multiples of 4



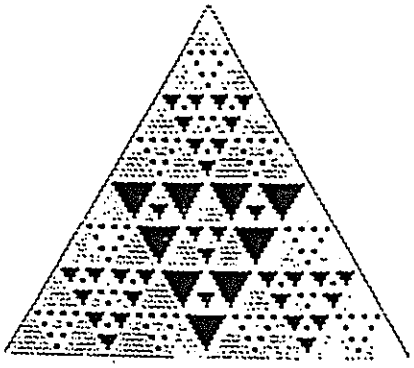
Multiples of 5



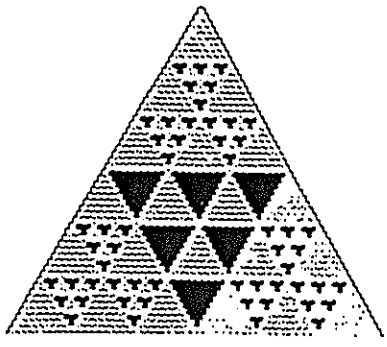
Multiples of 6



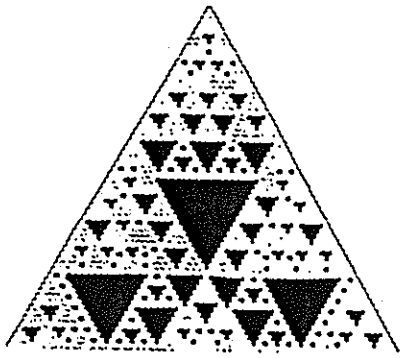
Multiples of 7



Multiples of 8



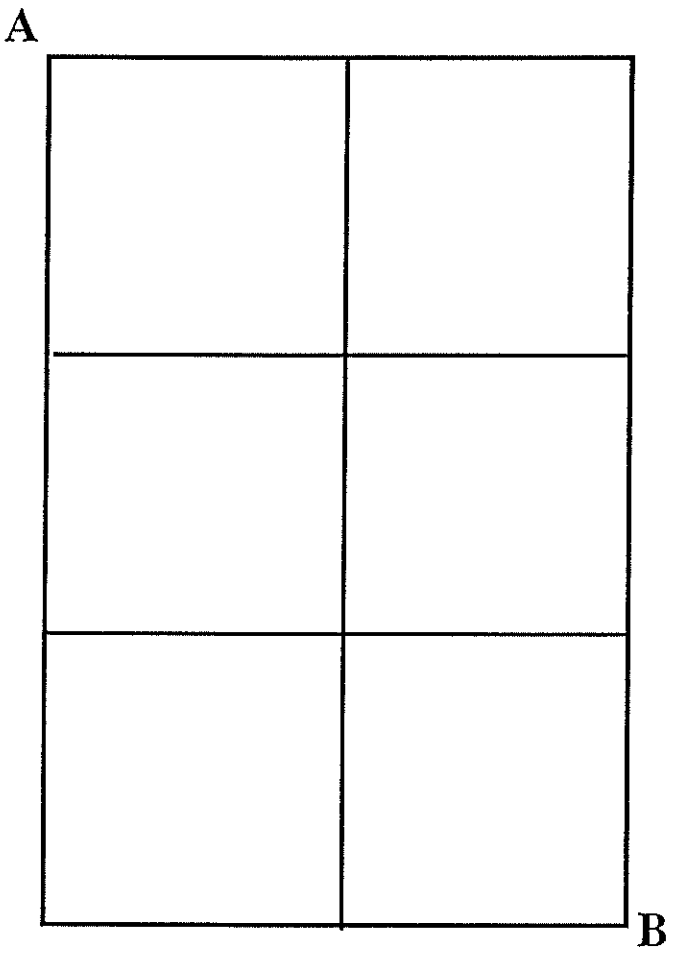
Multiples of 9



Multiples of 10

# NUMBER OF PATHS ON A GRID

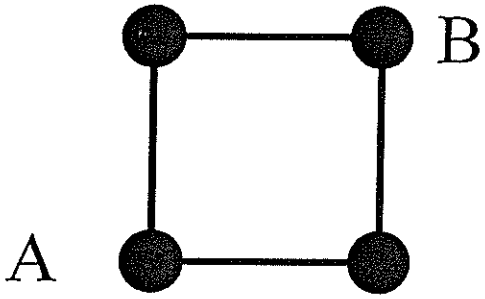
How many total paths are there from A to B on the grid below? Each move must go East (right) or South (down).



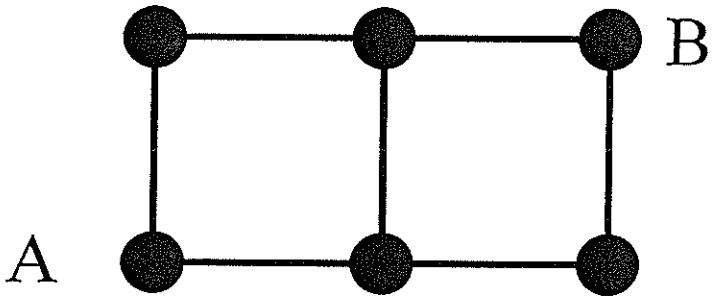

 

# Number of paths on a small grid

How many paths are there from A to B on the grid below?
(Assume that you always go North or East.)

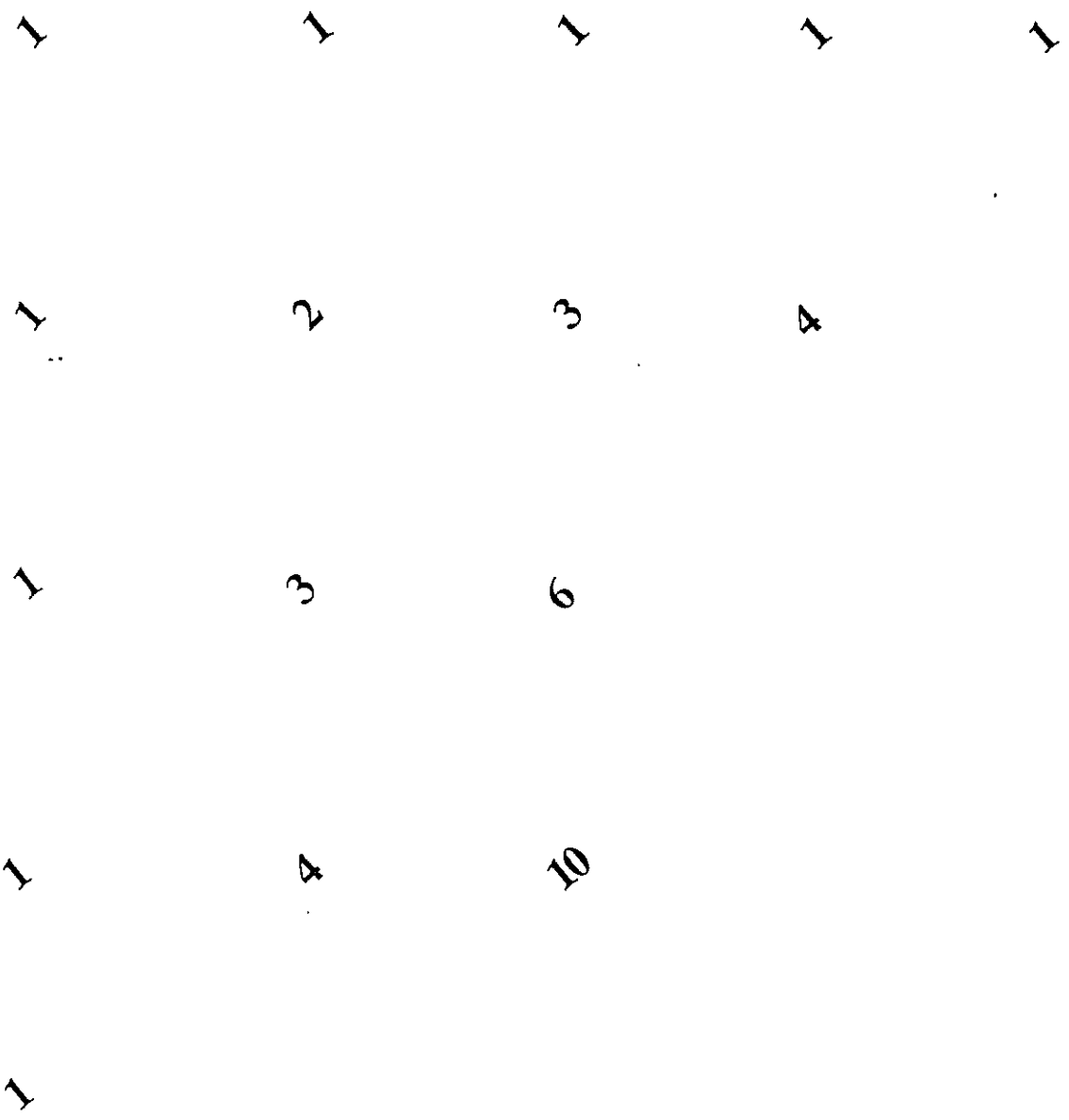



How many paths are there from A to B on the grid below?
(Assume that you always go North or East.)



# GRID OVERLAY



# THE HANDSHAKE PROBLEM

Suppose there are 8 people in your group and each person shook hands with every person in the group. How many handshakes would there be altogether? Solving simpler problems and making a T-Chart can help you solve the problem by finding a pattern.

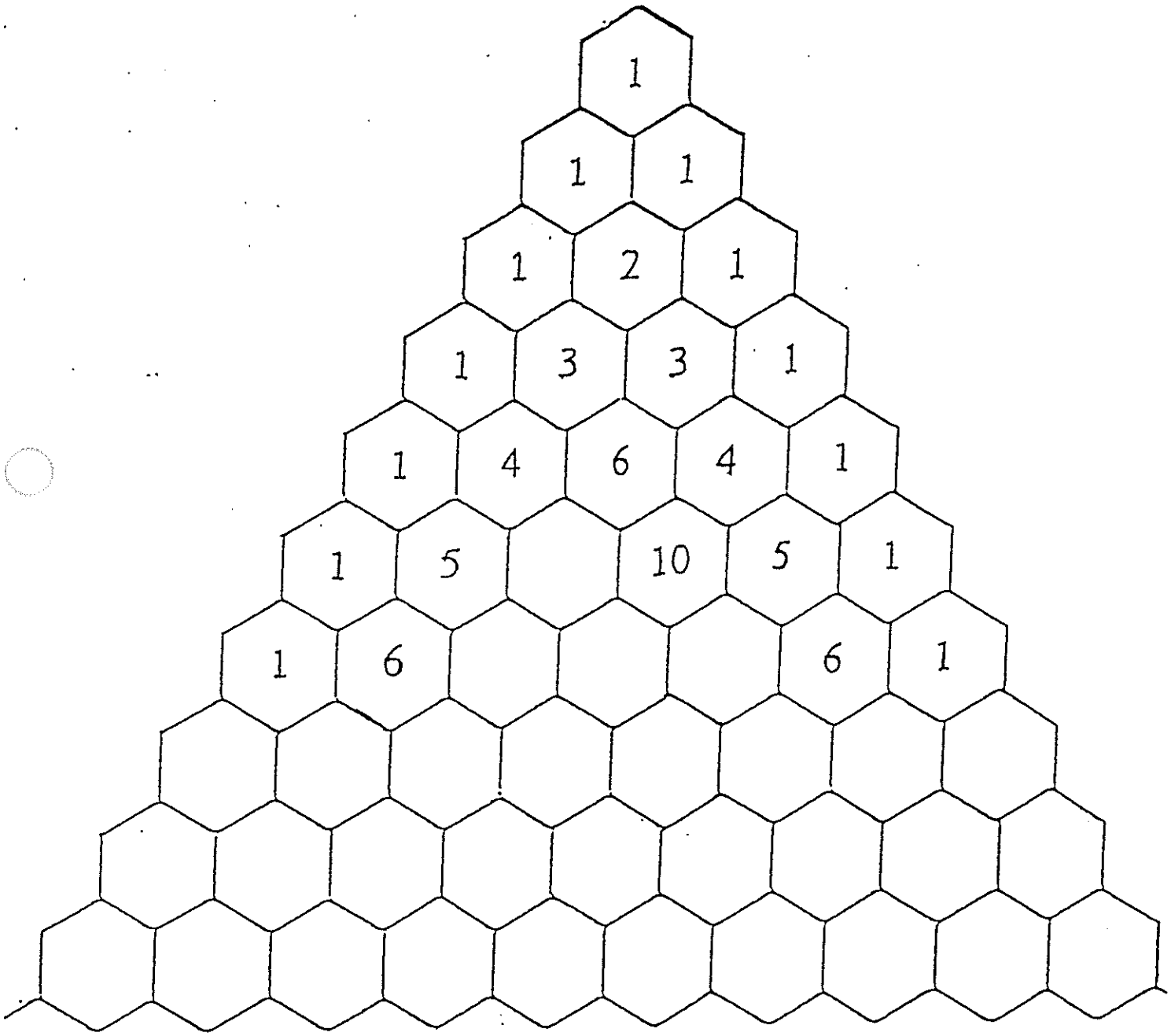
If there was 1 person, there would be 0 handshakes. With 2 people, there would be 1 handshake. How many handshakes with 3 people? 4 people? 5 people? Fill in the chart below to find the pattern.

| Number of People | Total # of Handshakes |
|---|---|
| 1 | |
| 2 | ____ |
| 3 | ____ |
| 4 | ____ |
| 5 | ____ |
| 6 | ____ |
| 7 | ____ |
| 8 | ____ |
| | ____ |

# PASCAL'S TRIANGLE

**Complete the next two rows of Pascal's Triangle.**



# CATS

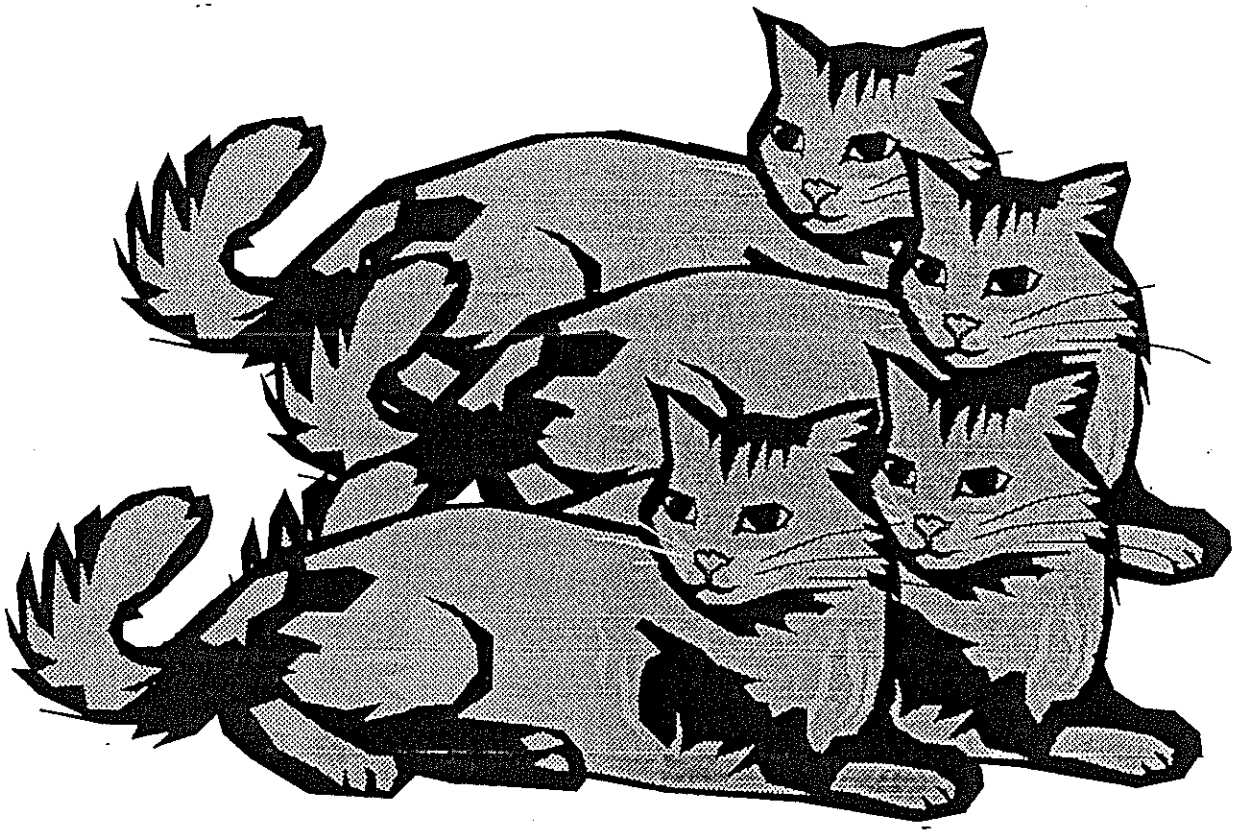
**How many ways are there to spell CATS?**

C

A A

T T T

S S S S



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| 6 | | |
| | | |

 

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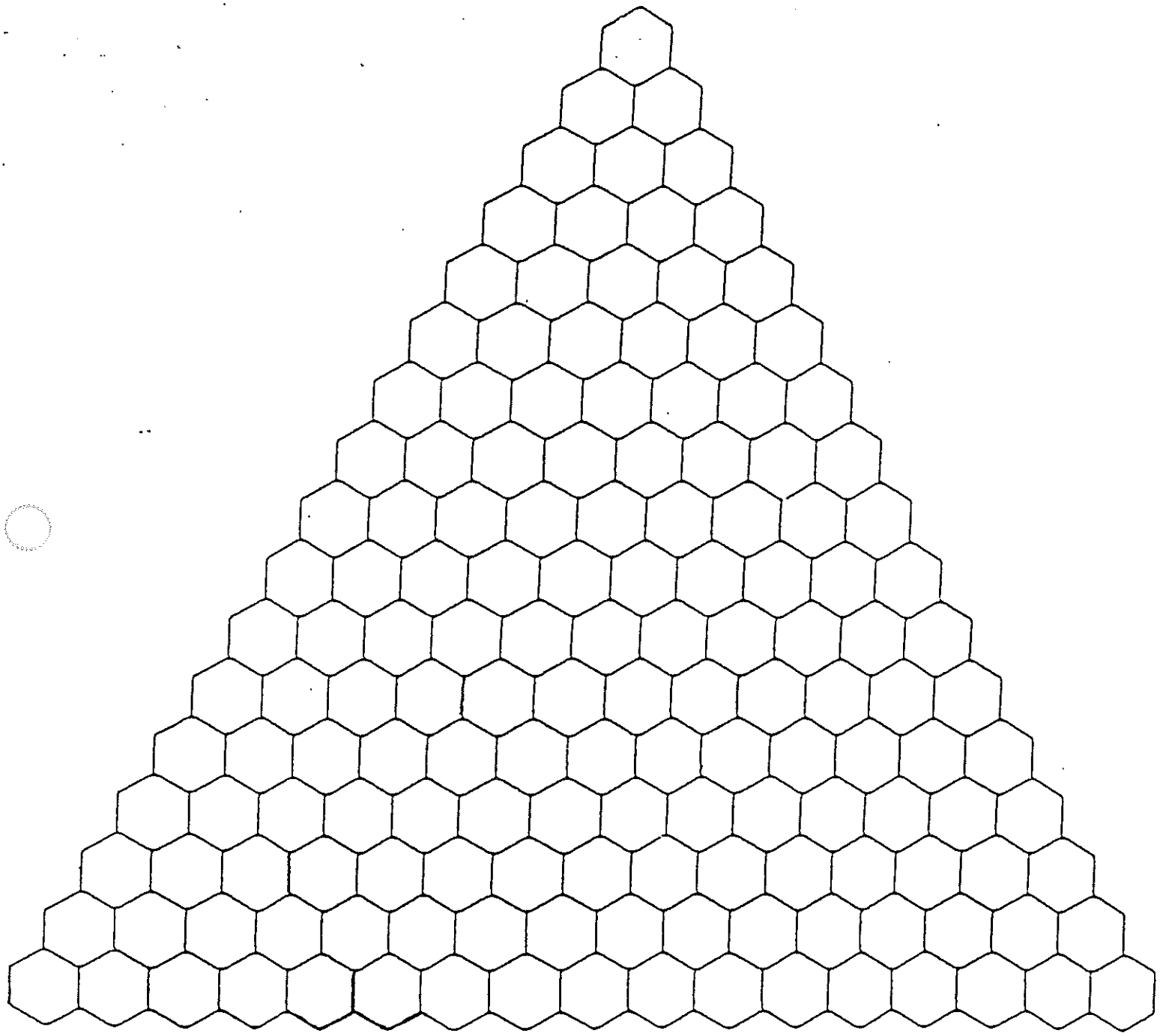

 

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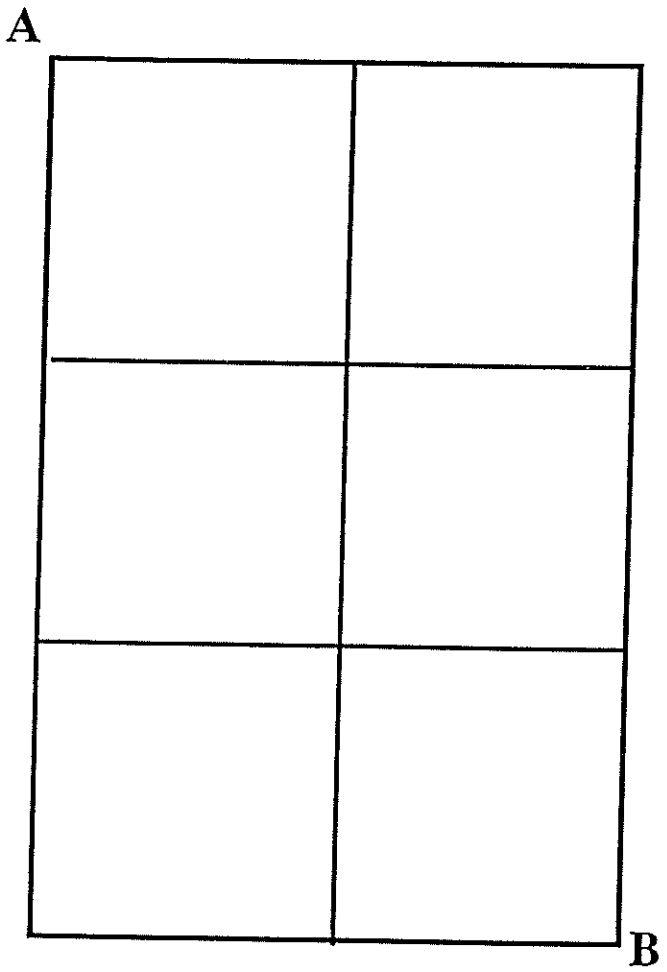
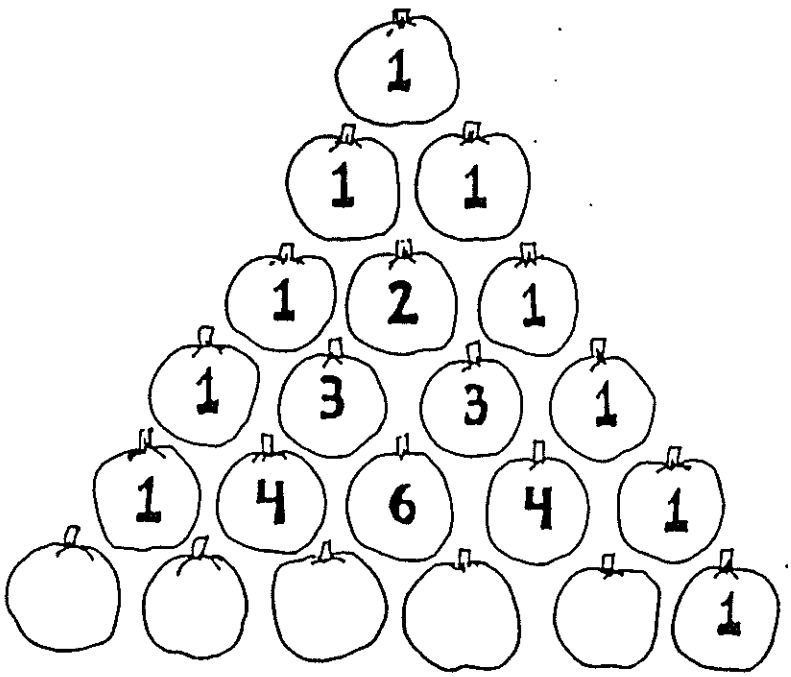


*Exploring Discrete Mathematics in the Classroom*

# *Powerful Patterns...Nifty Numbers K-4*

*A Workshop for Teachers About Patterns in Numbers*




*Developed by Joseph G. Rosenstein and Valerie A. DeBellis*

*in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Linda Bello, Carol Jacobs, Ann Lawrence, and Karen Wheeler*

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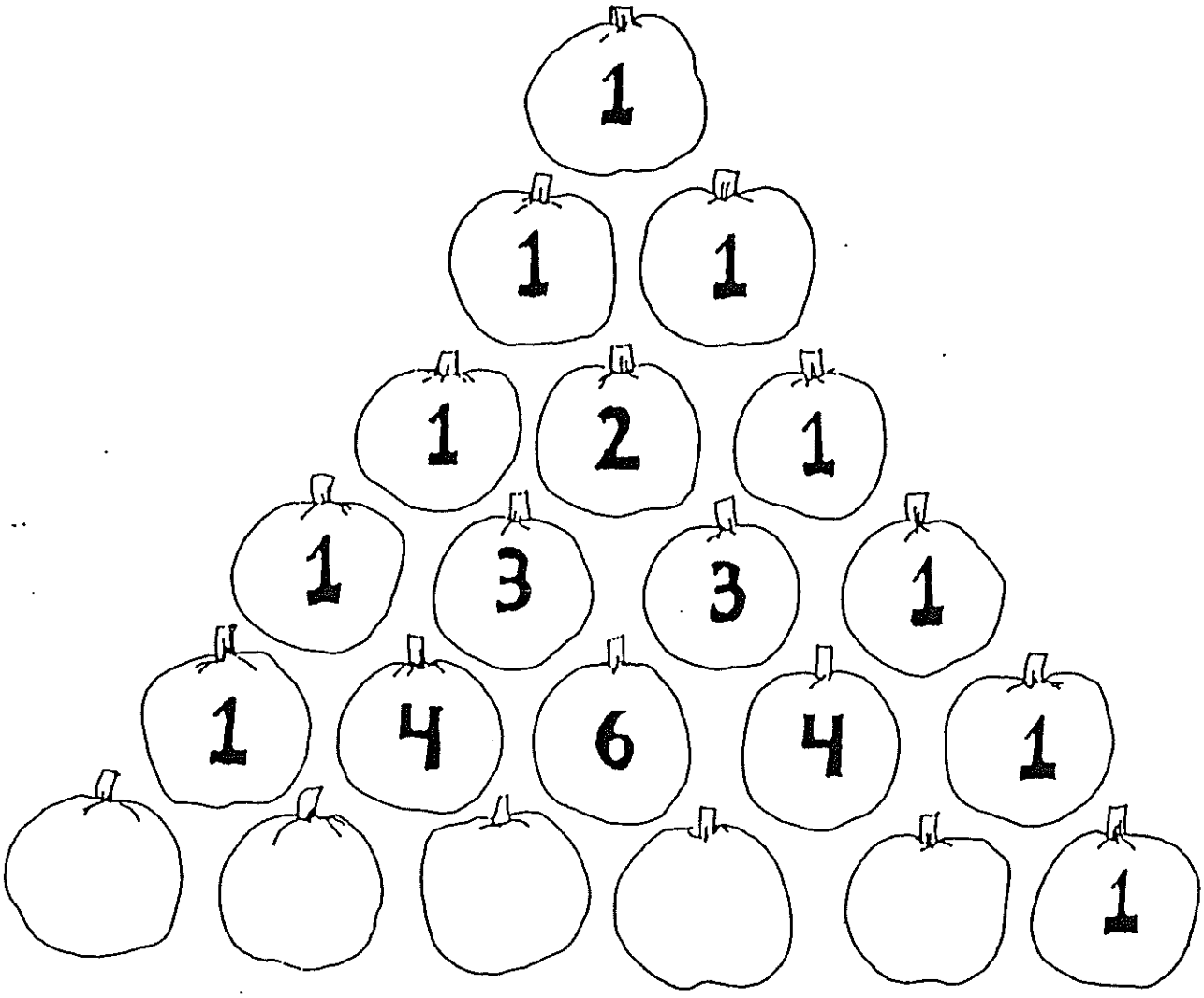
| 1 | |
| 2 | ____ |
| 3 | ____ |
| 4 | ____ |
| 5 | ____ |
| 6 | ____ |
| 7 | ____ |
| 8 | ____ |
| | ____ |

# PASCAL'S PUMPKINS




**What numbers belong on the bottom row of pumpkins?**

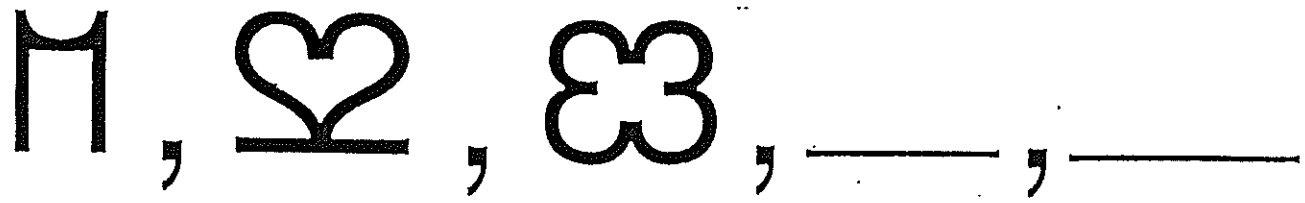
**Add the numbers in each row. What kind of pattern do you see?**

**What other patterns can you find in Pascal's Pumpkin Patch?**

# SYMBOLIC PATTERNS

Find the next two terms in each pattern.



**Amy, Bart, Carol, _____ , _____**

**One, Four, Three, _____ , _____**

# CATS

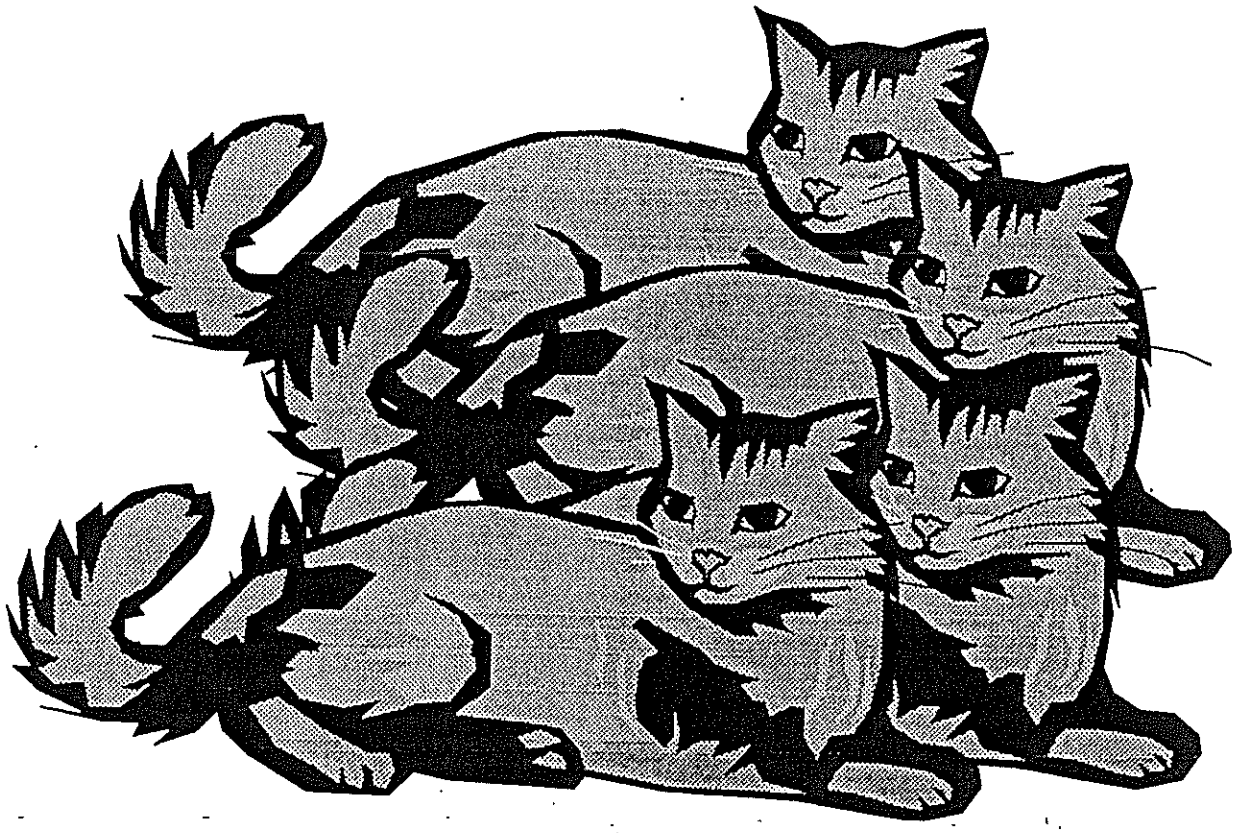
**How many CATS do you see?**

C

A A

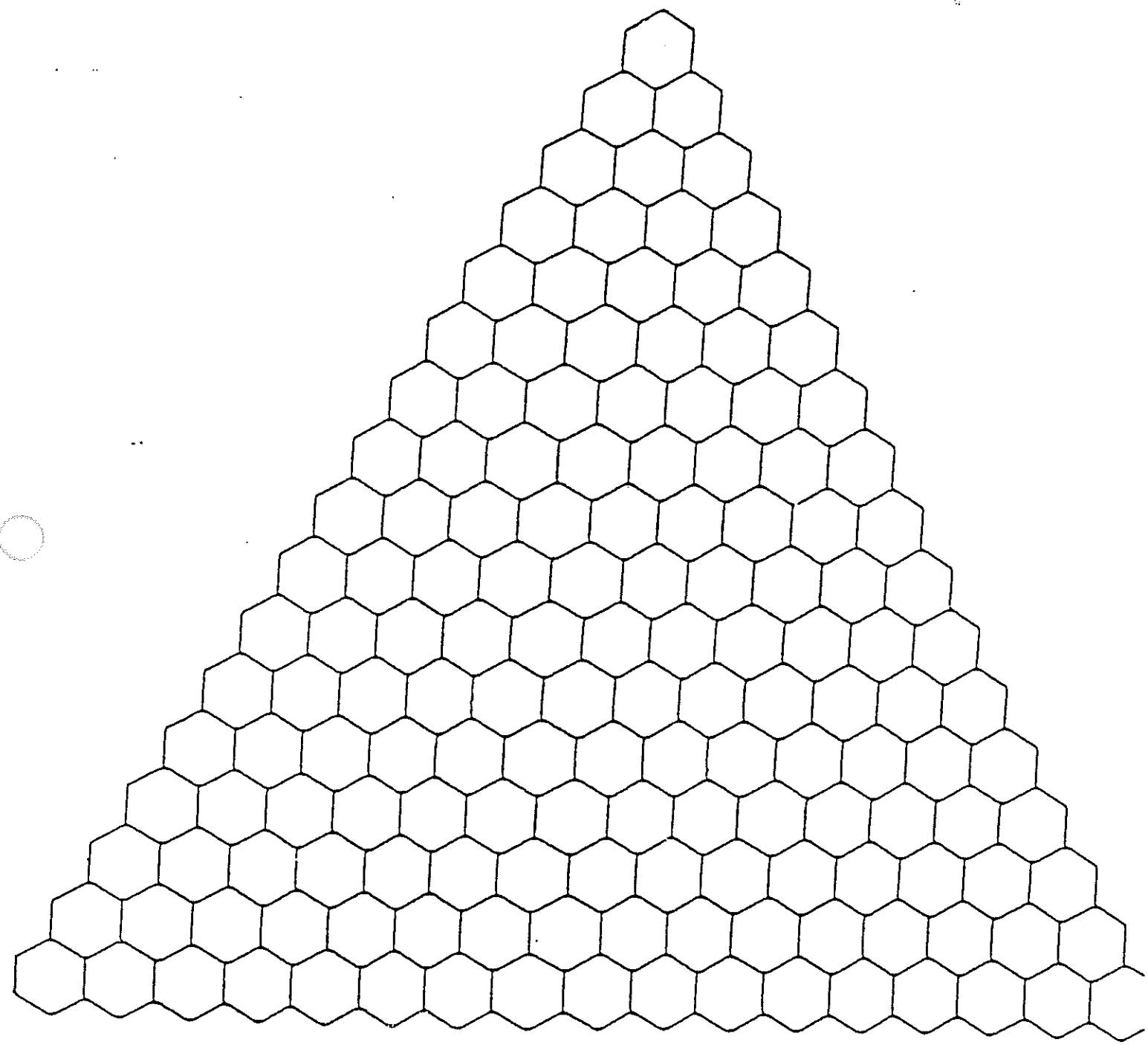
T T T

S S S S



# COLORFUL PATTERNS IN PASCAL'S TRIANGLE



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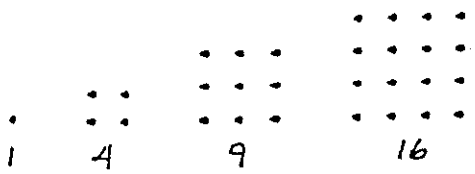
| Number of Animals | Ways to Board | Total # of Ways |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| | | |

# DOT AND NUMBER PATTERNS

1. Sketch the next two dot figures in the pattern below. Count the dots in each figure and record your answer. Extend the number pattern.



1 4 9 16

2. Describe the number pattern. _____

3. Does this number pattern continue? _____
   Try another figure to see.

4. Use your results to fill in the missing numbers in the following pattern.
   For each step, write the number of dots added to the previous figure.

1

4 = 1 + 3

9 = 1 + 3 + ___

16 = 1 + 3 + ___ + ___

_____ = ___ + ___ + ___ + ___ + ___

_____ = ___ + ___ + ___ + ___ + ___ + ___

5. Sketch the next three dot triangles in the pattern below. Count the dots in each figure and record your answer. Extend the number pattern.

1 3 6 10

6. Use your results to fill in the missing numbers in the following pattern.

1

3 = 1 + 2

6 = 1 + 2 + ___

10 = ___ + ___ + ___ + ___

_____ = ___ + ___ + ___ + ___ + ___

_____ = ___ + ___ + ___ + ___ + ___ + ___ + ___

7. Describe this pattern. _____

# PATTERNS AND RULES

**Find each pattern. Then complete the table and rule.**

1.

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 14 | 15 | 16 | ___ | ___ | ___ | ___ |

Rule:
Add 9.

2.

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 20 | 15 | 7 | 12 | 50 | 100 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 9 | 16 | ___ | ___ | ___ | ___ | ___ |

Rule: _____

3.

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 17 | 9 | 11 | 25 | 31 | 100 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 10 | 34 | 18 | ___ | ___ | ___ | ___ |

Rule: _____

4.

| | | | | | | |
|---|---|---|---|---|---|---|
| 9 | 5 | 15 | 8 | 11 | 13 | 100 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 45 | 25 | 75 | ___ | ___ | ___ | ___ |

Rule: _____

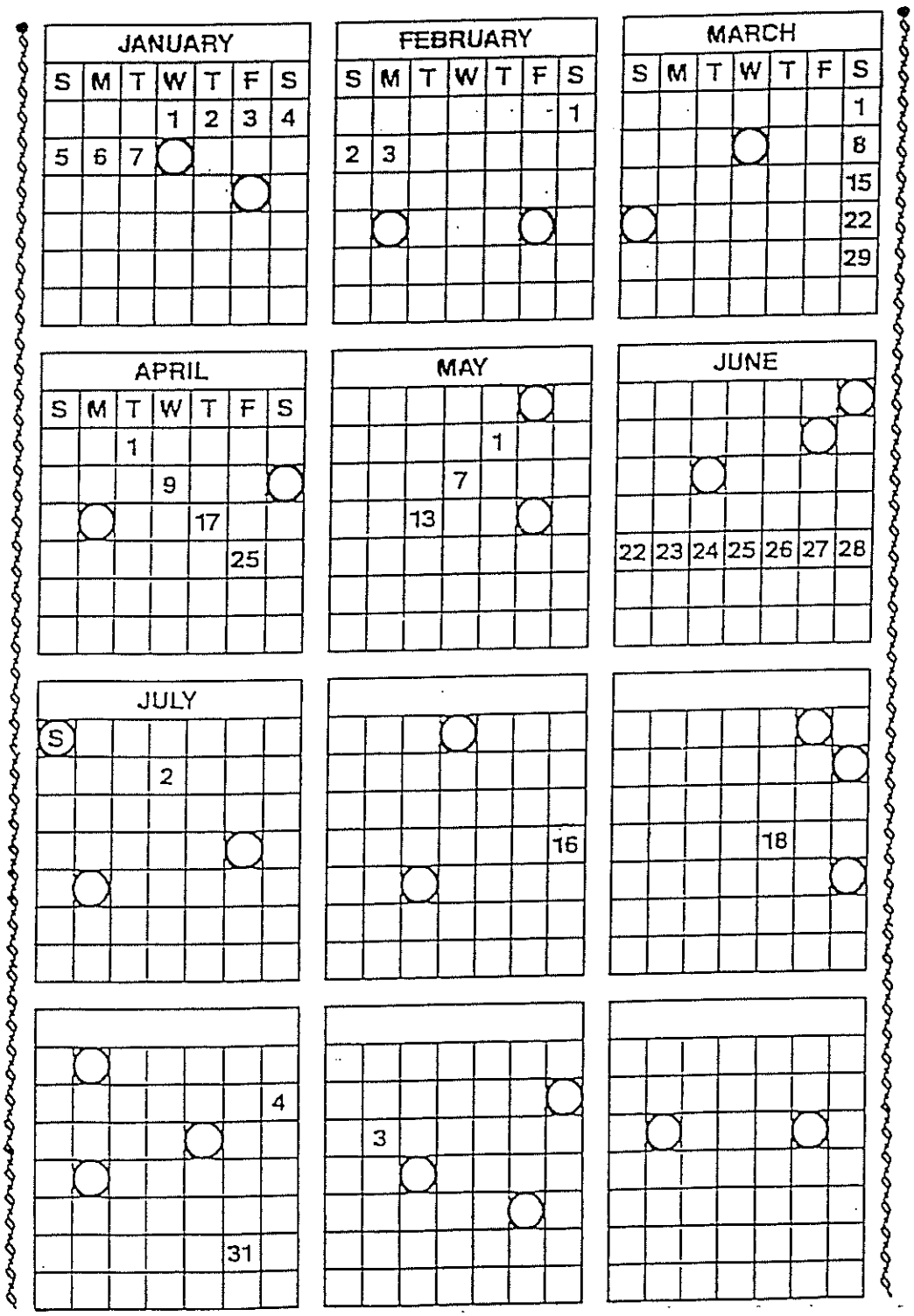
5.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 3 | 5 | 7 | 9 | ___ | ___ | ___ |

Rule: _____

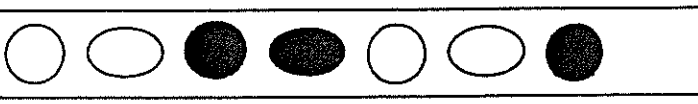
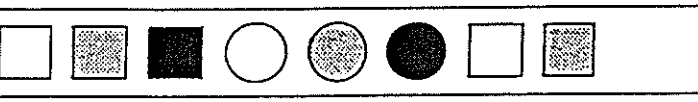
# CALENDAR PATTERNS

**Here is a 12-month calendar. Most of the numbers, days, and months are missing. Fill in the names of the missing months. Fill in the circled squares.**



# SHAPE PATTERNS

**Find the pattern in each row. Draw shapes that continue the pattern.**

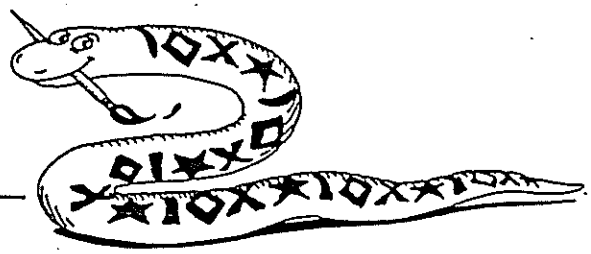
1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 

Explain one of the patterns here:

# COUNT AND CHECK

**Find a pattern, then continue it. Any pattern is fine----if you can explain it.**



1. 1 2 3 1 2 3 1 2 3 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

2. 1 2 1 1 2 1 1 1 2 1 1 1 1 2 ___ ___ ___ ___ ___ ___

3. 5 1 5 5 1 5 5 5 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

4. 3 6 4 5 3 6 4 5 3 6 4 5 ___ ___ ___ ___ ___ ___ ___ ___

5. 0 1 0 0 2 0 0 0 3 0 0 0 0 4 ___ ___ ___ ___ ___ ___

6. 3 9 3 3 8 3 3 3 7 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

7. 1 2 3 1 3 4 1 4 5 1 5 6 ___ ___ ___ ___ ___ ___ ___ ___

8. 0 9 1 1 8 2 2 2 7 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

9. 9 0 8 8 1 7 7 7 2 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

10. 1 1 2 1 2 3 1 2 3 4 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

# PREDICTABLE PATTERNS

**Sonya Kovalevsky (1850-1891) was fascinated by infinite sequences. Follow her example by filling in the spaces to continue these sequences.**

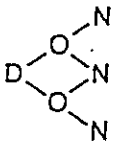
1. 1, 3, 5, 7, 9, _____, _____, _____, _____
2. 4, 8, 12, 16, 20, _____, _____, _____
3. 1, 2, 4, 7, 11, _____, _____, _____, _____
4. 1, 3, 6, 10, 15, _____, _____, _____
5. 1, 4, 9, 16, 25, _____, _____, _____, _____
6. 1, 2, 4, 8, 16, _____, _____, _____, _____
7. 3, 6, 9, 12, _____, _____, _____, _____
8. 3, 2, 6, 5, 15, 14, _____, _____, _____, _____
9. 1, 3, 2, 4, 3, _____, _____, _____, _____
10. 2, 3, 7, 16, 32, _____, _____
11. 1, 1, 2, 3, 5, 8, 13, 21, _____, _____, _____, _____
12. 3, 5, 9, 15, 23, 33, _____, _____, _____
13. 1, 2, 5, 10, 17, 26, _____, _____, _____
14. 1, 4, 9, 1, 6, 2, 5, 3, 6, 4, 9, 6, 4, _____, _____, _____, _____
15. 1, 1, 4, 3, 9, 6, 16, 10, 25, 15, _____, _____, _____, _____



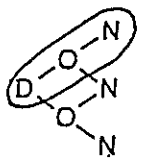

 

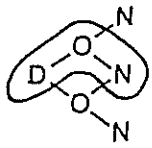
# LETTER COMBINATION PATTERNS

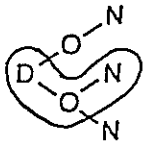
**Read each letter triangle from left to right. Use a different path to spell the same words. Circling the paths may help you find how many there are.**

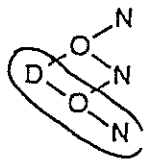
Example: 

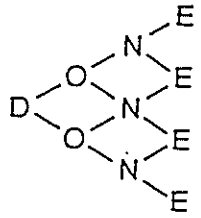

4 paths



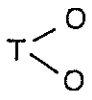



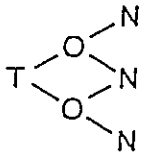




1. 

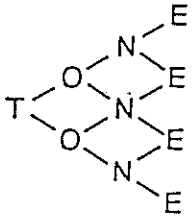

_____ paths

2. 

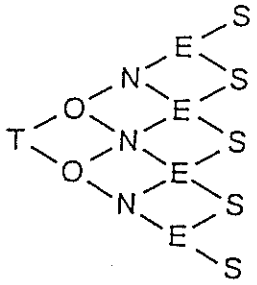

_____ paths

3. 


_____ paths

4. 


_____ paths

5. 


_____ paths

6. Make a letter puzzle of your first name.
How many paths can you take to spell it? _____

7. Summarize your results by completing the table.

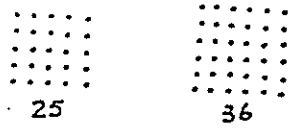
| Number of letters in each word | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Number of paths | 2 | 4 | | | | | |

# Answer Keys For Activities

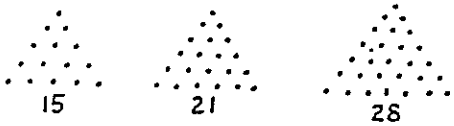
## Dot and Number Patterns

1.




2. numbers squared 3. yes 4. $9 = 1+3+5$
$16 = 1+3+5+7$ $25 = 1+3+5+7+9$
$36 = 1+3+5+7+9+11$

5.




6. $6 = 1+2+3$ $10 = 1+2+3+4$
$15 = 1+2+3+4+5$ $21 = 1+2+3+4+5+6$
7. Triangular numbers: add the next consecutive number to the previous sum

**Answer Key: TKHM #7 (Patterns k-4)**

## Patterns and Rules

1.

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 |

Rule: Add 9.

2.

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 20 | 15 | 7 | 12 | 50 | 100 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 9 | 16 | 11 | 3 | 8 | 46 | 96 |

Rule: Subtract 4.

3.

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 17 | 9 | 11 | 25 | 31 | 100 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 10 | 34 | 18 | 22 | 50 | 62 | 200 |

Rule: Multiply by 2.

4.

| | | | | | | |
|---|---|---|---|---|---|---|
| 9 | 5 | 15 | 8 | 11 | 13 | 100 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 45 | 25 | 75 | 40 | 55 | 65 | 500 |

Rule: Multiply by 5.

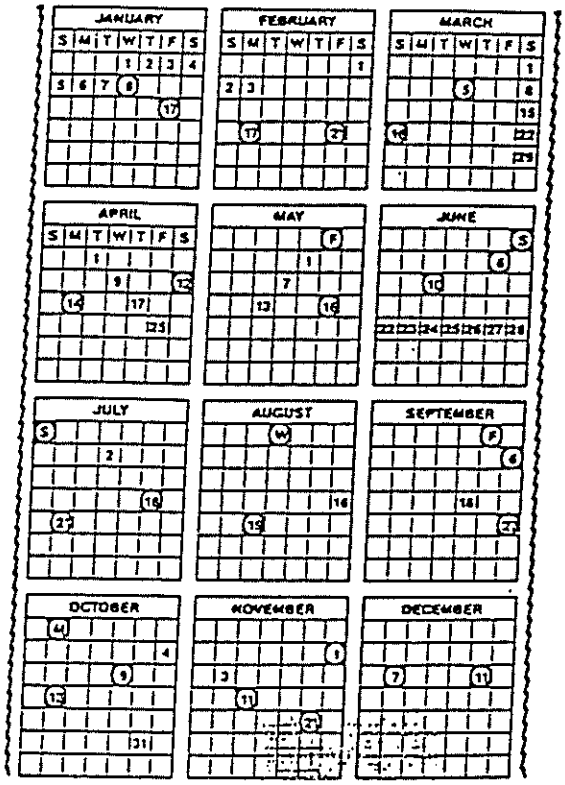
5.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| 3 | 5 | 7 | 9 | 11 | 13 | 15 |

Rule: Multiply by 2 and add 1.

**Answer Key: TKHM #8 (Patterns k-4)**

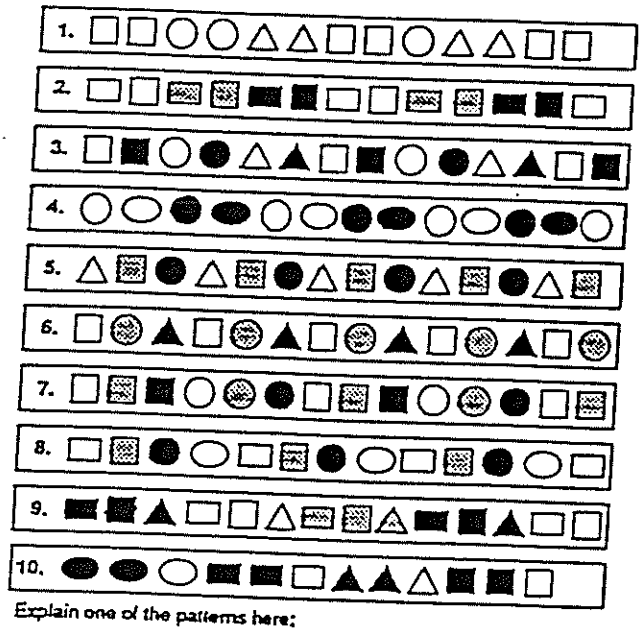
## Calendar Patterns




**Answer Key: TKHM #9 (Patterns k-4)**

## Shape Patterns

Answers may vary. The most likely answers are:




Explain one of the patterns here:

Answers will vary. For example: In problem 3, there are two of every shape, and the second shape of each pair is black. Also, the sequence of shapes runs squares, circles, triangles.

**Answer Key: TKHM #10 (Patterns k-4)**

# Answer Keys For Activities

### Count and Check

Answers may vary. The most likely answers are:

| | |
|---|---|
| 1. | 1 2 3 1 2 3 1 2 3 1 2 3 1 2 3 1 2 3 1 |
| 2. | 1 2 1 1 2 1 1 1 2 1 1 1 1 2 1 1 1 1 1 2 |
| 3. | 5 1 5 5 1 5 5 5 1 5 5 5 5 1 5 5 5 5 |
| 4. | 3 6 4 5 3 6 4 5 3 6 4 5 3 6 4 5 3 6 4 5 |
| 5. | 0 1 0 0 2 0 0 0 3 0 0 0 0 4 0 0 0 0 0 5 |
| 6. | 3 9 3 3 6 3 3 3 7 3 3 3 3 6 3 3 3 3 3 |
| 7. | 1 2 3 1 3 4 1 4 5 1 5 6 1 6 7 1 7 8 1 8 |
| 8. | 0 9 1 1 8 2 2 2 7 3 3 3 3 6 4 4 4 4 4 |
| 9. | 9 0 8 8 1 7 7 7 2 6 6 6 6 3 5 5 5 5 5 |
| 10. | 1 1 2 1 2 3 1 2 3 4 2 2 3 2 3 4 2 3 4 |

**Answer Key: TKHM #11 (Patterns k-4)**

### Predictable Patterns

Sonya Kovalevsky contributed to the study of infinite series and sequences. Seeing patterns and forming generalizations is an important part of this work. In this activity students have the experience of making discoveries and predicting some number sequences.

Solutions:

1. 11, 13, 15, 17
2. 24, 28, 32
3. 16, 22, 29, 37
4. 21, 28, 36
5. 36, 49, 64, 81
6. 32, 64, 128, 256
7. 15, 18, 21, 24
8. 42, 41, 123, 122 (subtract 1, times 3)
9. 5, 4, 6, 5
10. 57, 93 (increase by squares)
11. 34, 55, 89, 144 (Fibonacci sequence)
12. 45, 59, 75
13. 37, 50, 65
14. 8, 1, 1, 0 (square numbers with commas inserted)
15. 36, 21, 49, 28 (square and triangular numbers alternate)

**Answer Key: TKHM #12 (Patterns k-4)**

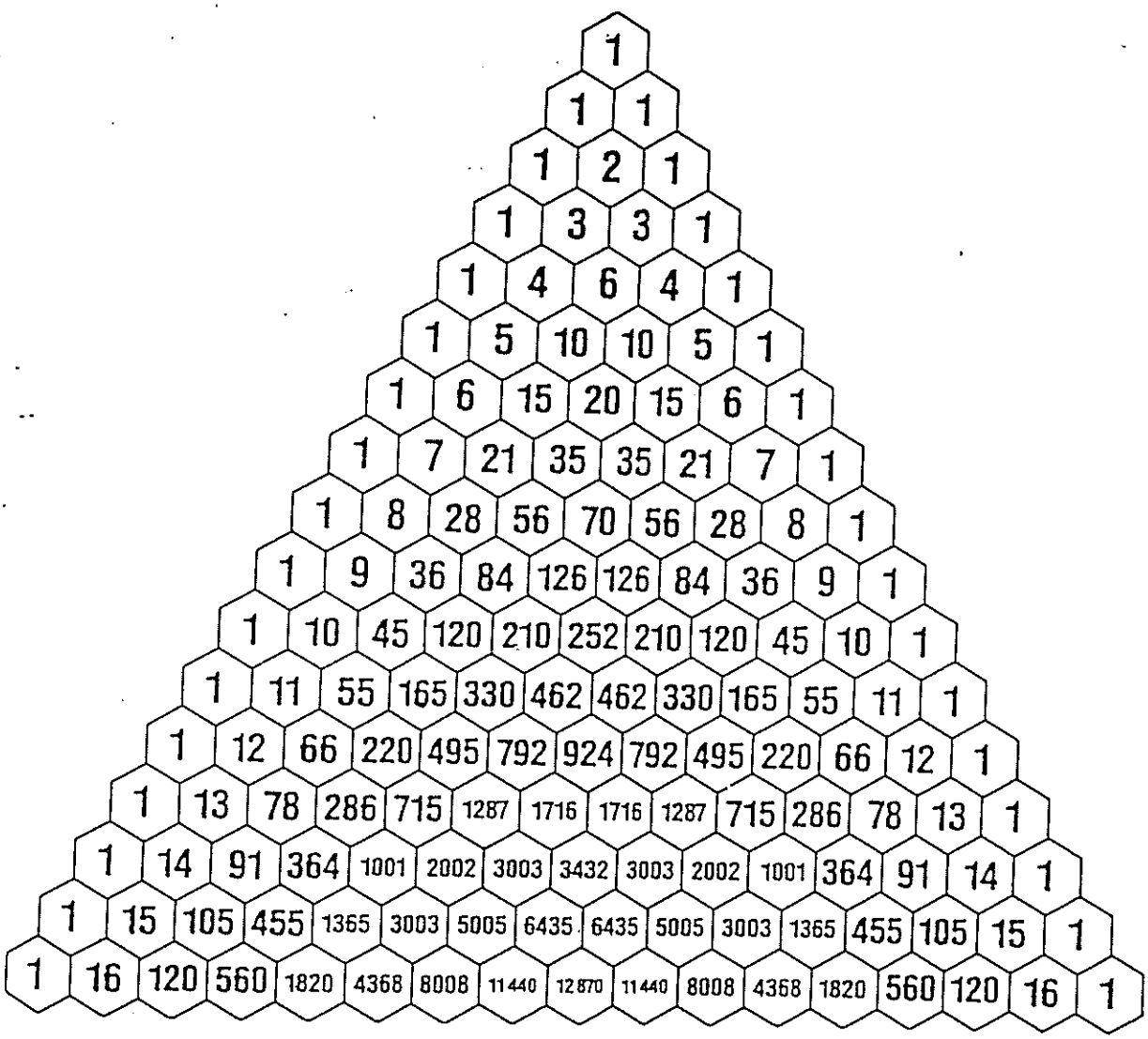
### Letter Combination Patterns

1. 8 paths 2. 2 paths 3. 4 paths 4. 8 paths
5. 16 paths 6. Answers will vary. Accept all correct student answers.
7.

| Number of letters in each word | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Number of paths | 2 | 4 | 8 | 16 | 32 | 64 | 128 |

**Answer Key: TKHM #13 (Patterns k-4)**

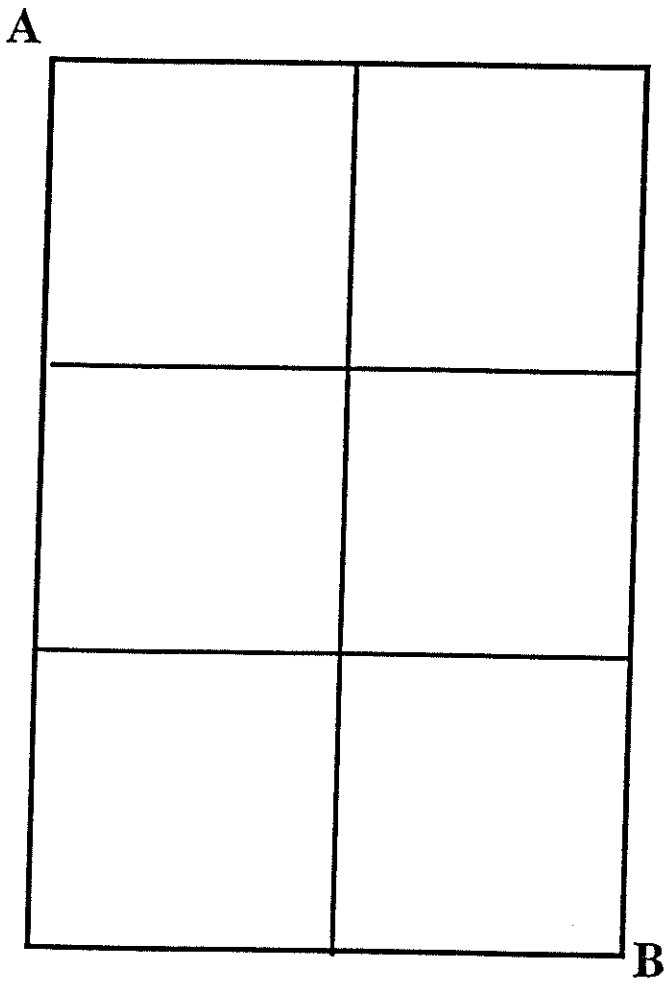
# PASCAL'S TRIANGLE



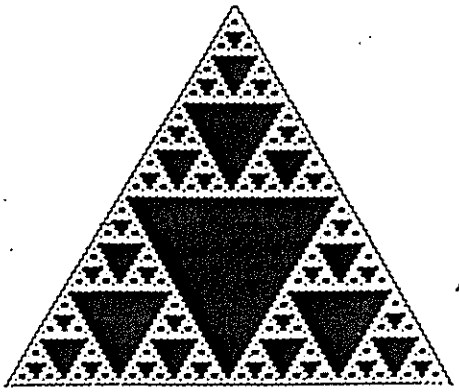
# NUMBER OF PATHS ON A GRID

**How many total paths are there from A to B on the grid below? Each move must go East (right) or South (down).**

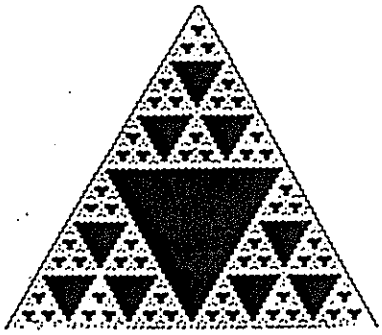


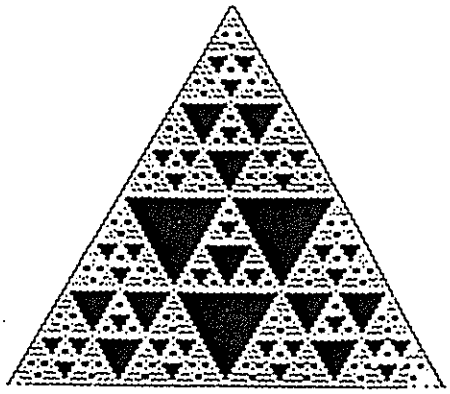
# VISUAL PATTERNS RESOURCE PAGE



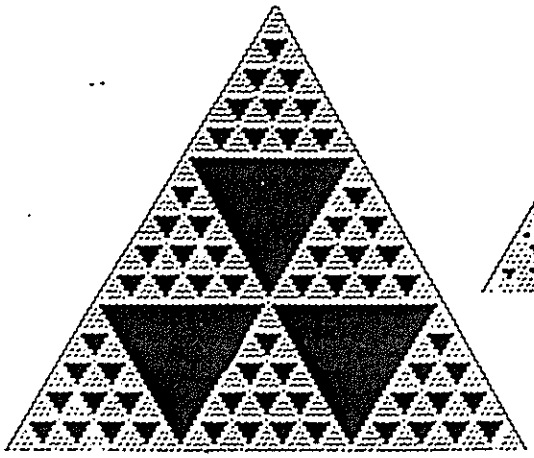
Multiples of 2



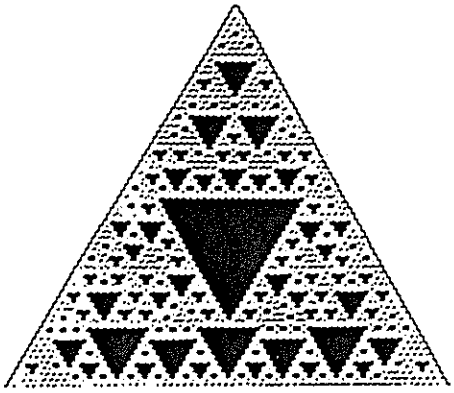
Multiples of 3



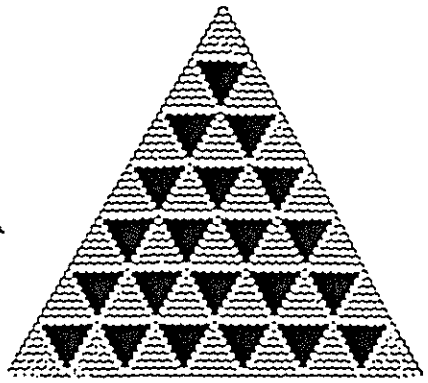
Multiples of 4



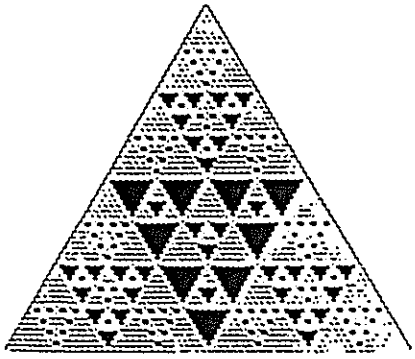
Multiples of 5



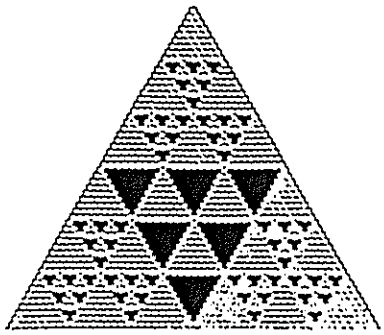
Multiples of 6



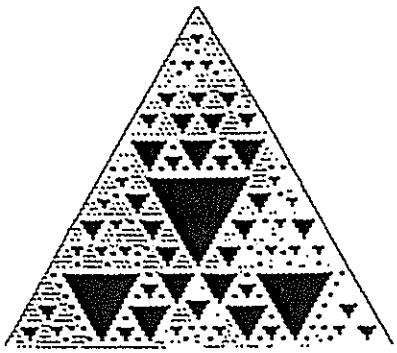
Multiples of 7



Multiples of 8



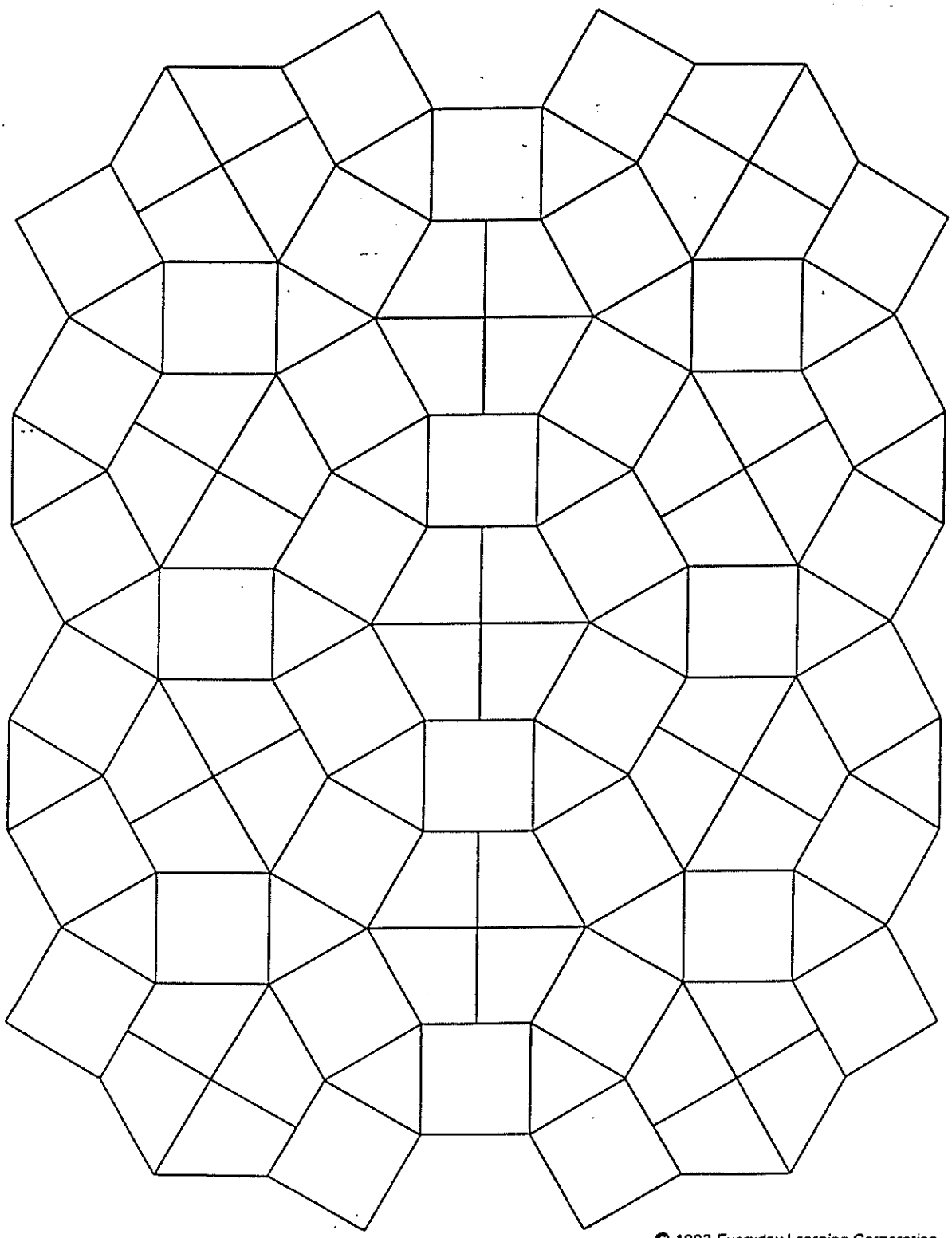
Multiples of 9



Multiples of 10

# HOME LINK

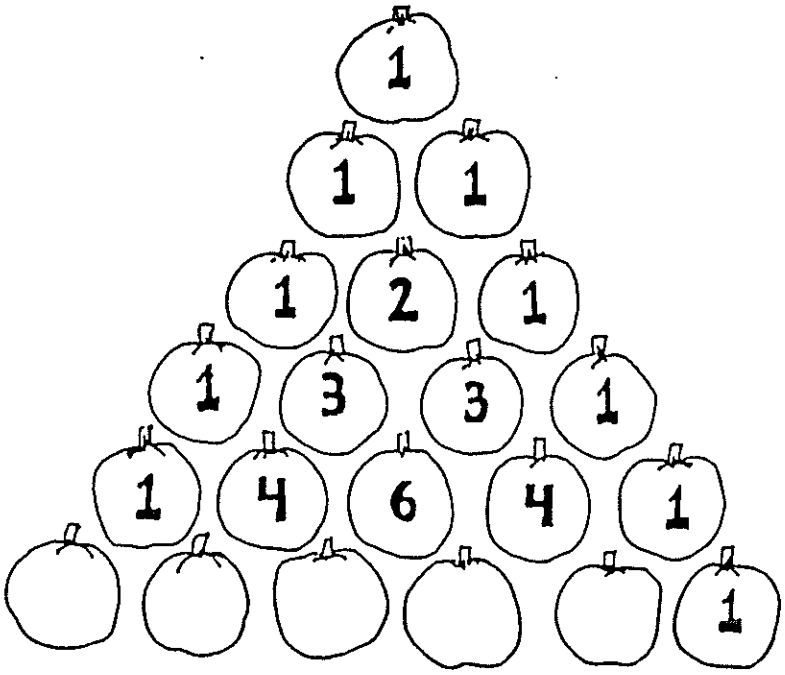
**Color the design. Use more than one color. Try to show a pattern.**



*Exploring Discrete Mathematics in the Classroom*

# *Powerful Patterns...Nifty Numbers K-4*

*A Workshop for Teachers About Patterns in Numbers*




*Developed by Joseph G. Rosenstein and Valerie A. DeBellis*

*in collaboration with the following participants in the Rutgers Leadership Program in Discrete Mathematics: Linda Bello, Carol Jacobs, Ann Lawrence, and Karen Wheeler*

# THE HANDSHAKE PROBLEM

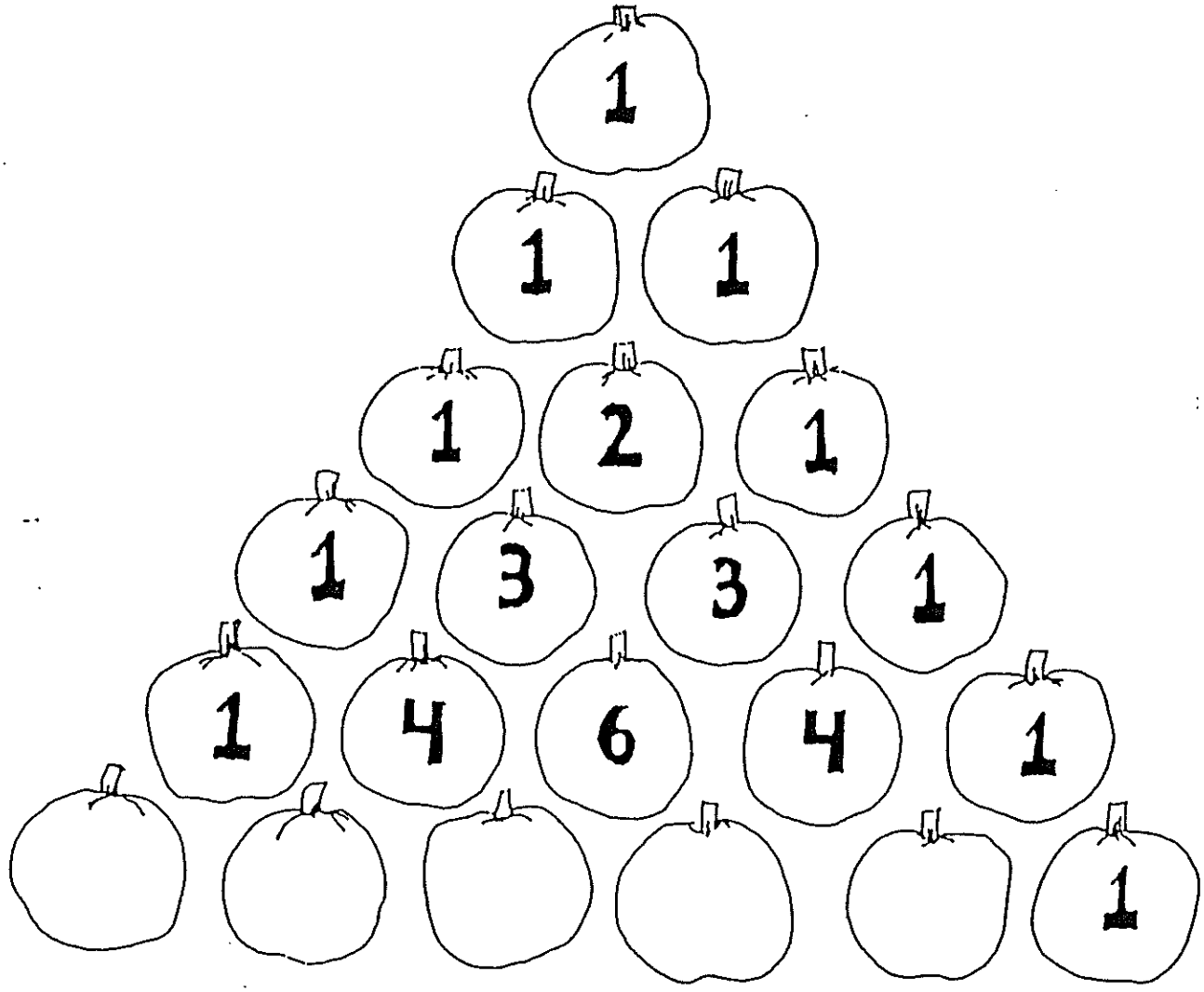
Suppose there are 8 people in your group and each person shook hands with every other person in the group. How many handshakes would there be altogether? Solving simpler problems and making a T-Chart can help you solve the problem by finding a pattern.

If there was 1 person, there would be 0 handshakes. With 2 people, there would be 1 handshake. How many handshakes with 3 people? 4 people? 5 people? Fill in the chart below to find the pattern.

| Number of People | Total # of Handshakes |
|---|---|
| 1 | |
| 2 | ____ |
| 3 | ____ |
| 4 | ____ |
| 5 | ____ |
| 6 | ____ |
| 7 | ____ |
| 8 | ____ |
| | ____ |

# PASCAL'S PUMPKINS



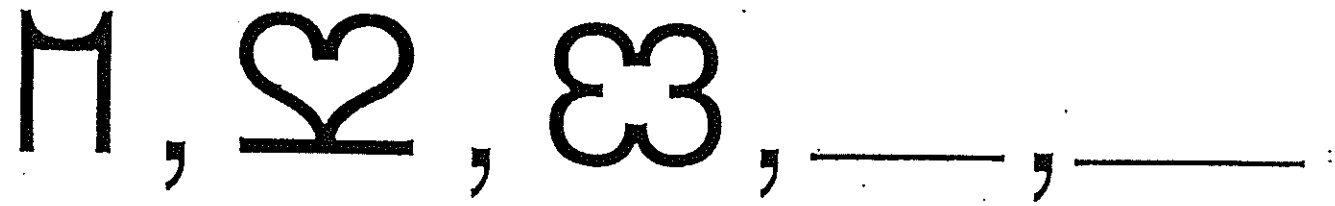

**What numbers belong on the bottom row of pumpkins?**

**Add the numbers in each row. What kind of pattern do you see?**

**What other patterns can you find in Pascal's Pumpkin Patch?**

# SYMBOLIC PATTERNS

**Find the next two terms in each pattern.**



**Amy, Bart, Carol, _____ , _____**

**One, Four, Three, _____ , _____**