Locally Private Release of Marginal Statistics

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Privacy with a coin toss

Perhaps the simplest possible formal privacy algorithm:

- Scenario. Each user has a single private bit of information
  - Encoding e.g. political/sexual/religious preference, illness, etc.

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- **Analysis.** Gives differential privacy with parameter $\varepsilon = \ln (p/(1-p))$
  - Works well in theory, but would anyone ever use this?

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- Local Differential privacy is state of the art in 2017: Randomized response invented in 1965: five decade lead time!

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Gender/Obese	0	1
0	0.28	0.22
1	0.29	0.21

Disease/Smoke	0	1
0	0.55	0.15
1	0.10	0.20

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- Need to design algorithms that minimize information per user
- First observation: a sampling trick
  - If we release n bits of information per user, the error is $n/\sqrt{N}$
  - If we sample 1 out of n bits, the error is $\sqrt{(n/N)}$
  - Quadratically better to sample than to share!

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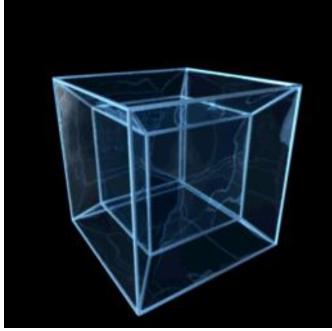
- If k is small (say, 2), and d is large (say 10s), Approach 1 is better
  - But there's another approach to try…

Hadamard transform



Instead of materializing the data, we can transform it

- Via Hadamard transform (the discrete Fourier transform for the binary hypercube)
  - Simple and fast to apply

$$\begin{bmatrix} H^* & H^* \\ H^* & -H^* \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}.$$

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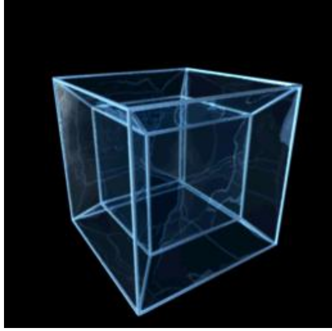

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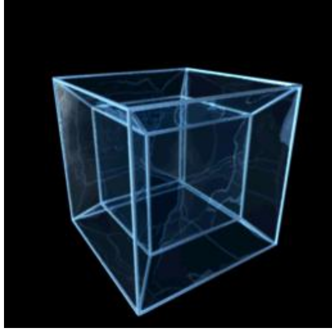

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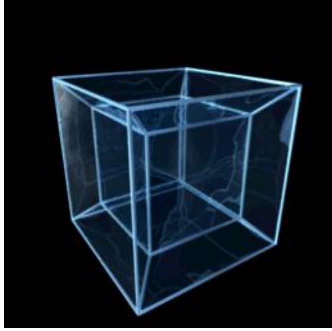

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- Property 1: only (d choose k) coefficients are needed to build any k-way marginal
  - Reduces the amount of information to release
- Property 2: Hadamard transform is a linear transform
  - Can estimate global coefficients by sampling and averaging
- Yields error proportional to $2^{k/2}d^{k/2}/\sqrt{N}$
  - Better than both previous methods (in theory)

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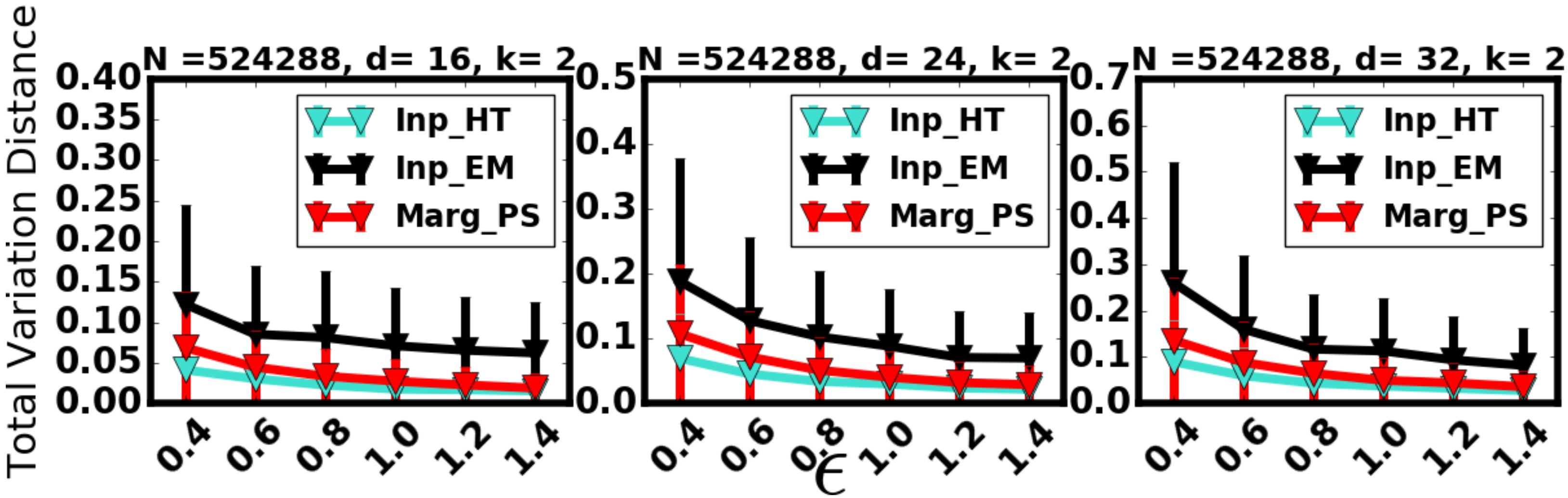
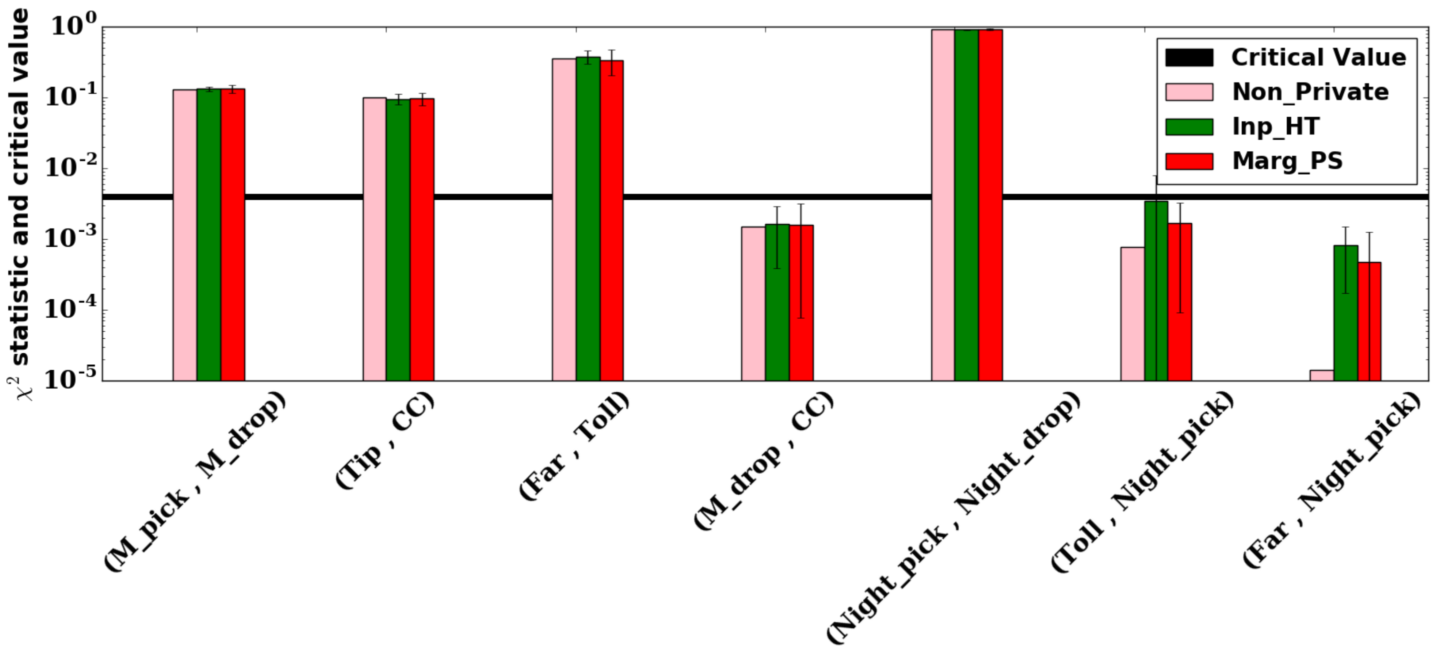
Empirical behaviour



- Compare three methods: Hadamard based (Inp_HT), marginal materialization (Marg_PS), Expectation maximization (Inp_EM)
- Measure sum of absolute error in materializing 2-way marginals
- N = 0.5M individuals, vary privacy parameter ε from 0.4 to 1.4

Applications – χ-squared test



- Anonymized, binarized NYC taxi data
- Compute χ-squared statistic to test correlation
- Want to be same side of the line as the non-private value!

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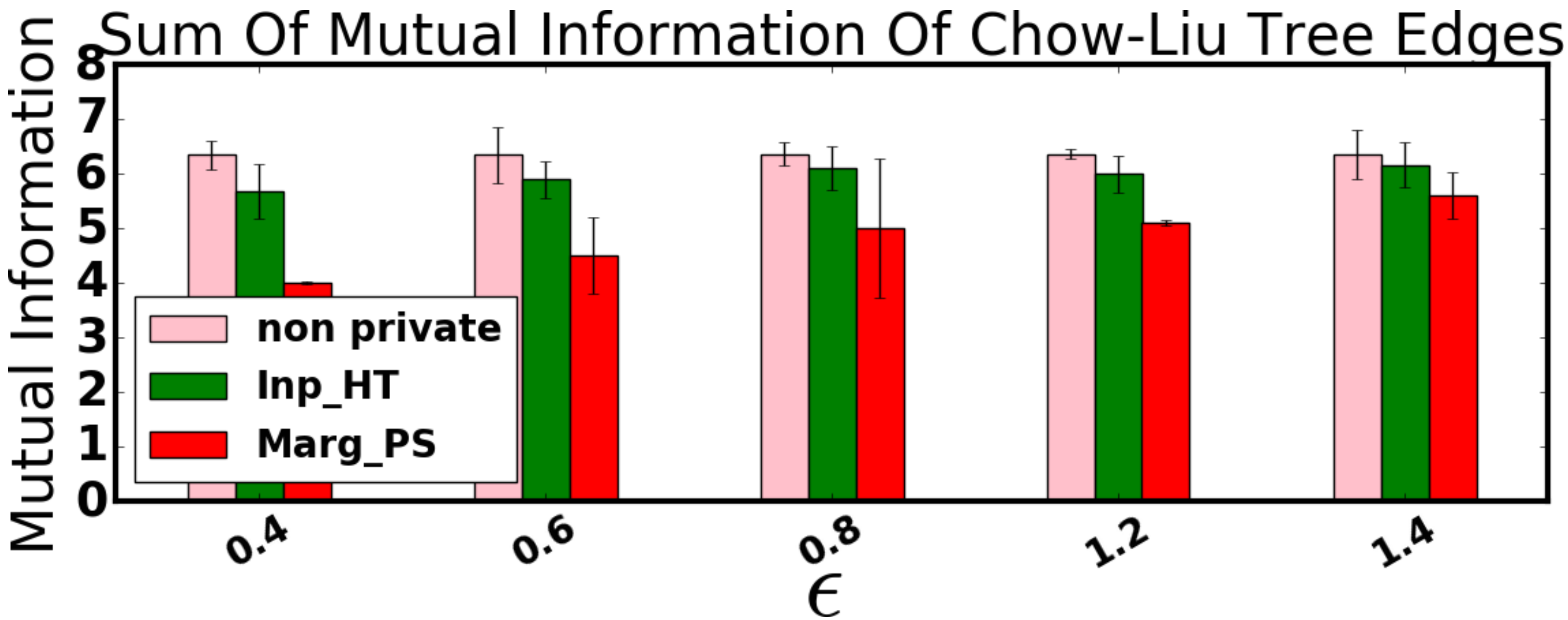
Application – building a Bayesian model



- Aim: build the tree with highest mutual information (MI)
- Plot shows MI on the ground truth data for evaluation purposes

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