Set Cover Algorithms For Very Large Datasets

Graham Cormode

Howard Karloff

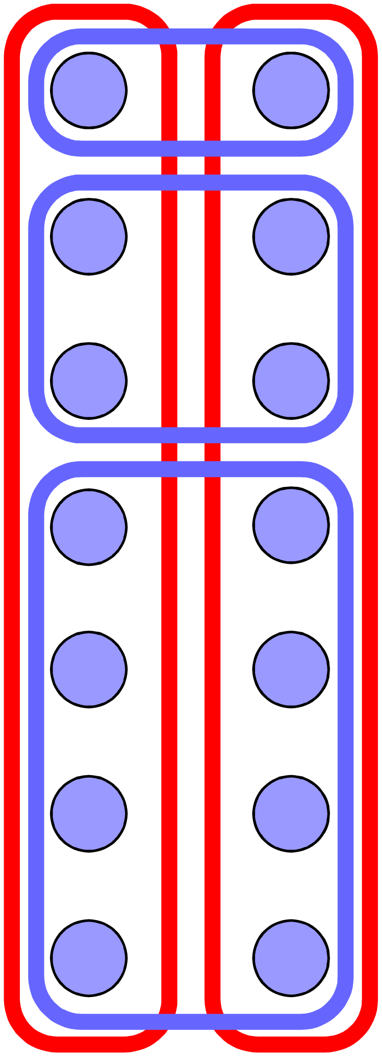
AT&T Labs-Research

Tony Wirth

University of Melbourne

Set Cover?

- Given a collection of sets over a universe of items
- Find smallest subcollection of sets that also cover all the items.



Why Set Cover?

- The set cover problem arises in many contexts:
  - Facility location: facility covers sites
  - Machine learning: labeled example covers some items
  - Information Retrieval: each document covers set of topics
  - Data mining: finding a minimal ‘explanation’ for patterns
  - Data quality: find a collection of rules to describe structure

How to solve it?

- Set Cover is NP-hard!
- Simple greedy algorithm:
  - Repeatedly select set with most uncovered items.
  - Logarithmic factor guarantee: $1 + \ln n$
  - No factor better than $(1 - o(1)) \ln n$ possible
- In practice, greedy very useful:
  - Better than other approximation algorithms
  - Often within 10% of optimal

Existing Algorithms

- Greedy algorithm: $1+\ln n$ approximation
  - Until all n elements of X are in C (initially empty):
    - Choose (one of) set(s) with maximum value of $|S_i - C|$
    - Let $C = C \cup S_{i^*}$
- Naïve algorithm: no guaranteed approximation
  - Sort the sets by their (initial) sizes, $|S_i|$, descending
  - Single pass through the sorted list:
    - If a set has an uncovered item, select it
    - Update C

Example greedy

ABCDE	ABDFG
AFG	BCG
GH	EH
C I	A
E	I

Optimum

ABCDE	ABDFG
AFG	BCG
GH	EH
CI	A
E	I

What's wrong?

- Try implementing greedy on large dataset:
  - Scales very poorly
- Millions of sets with universe of many millions of items?
- Dataset growth exceeds fast memory growth
- If forced to use disk: selecting "largest" set requires updating set sizes to account for covered items
- Even 30Mb instance required >1 minute to run on disk

Implementing greedy

- Main step: find set with largest $|S_i - C|$ value
- Inverted index:
  - Maintain updated sizes in priority queue
  - Inverted index records which sets each item is in
  - Costly to build index, no locality of reference
- Multipass solution:
  - Loop through all sets, calculating $|S_i - C|$ on the fly
  - Good locality of reference, but many passes!
  - If $|S_{i^*} - C|$ drops below a threshold:
    - Loop adds all sets with specific $|S_{i^*} - C|$ value

Idea for our algorithm

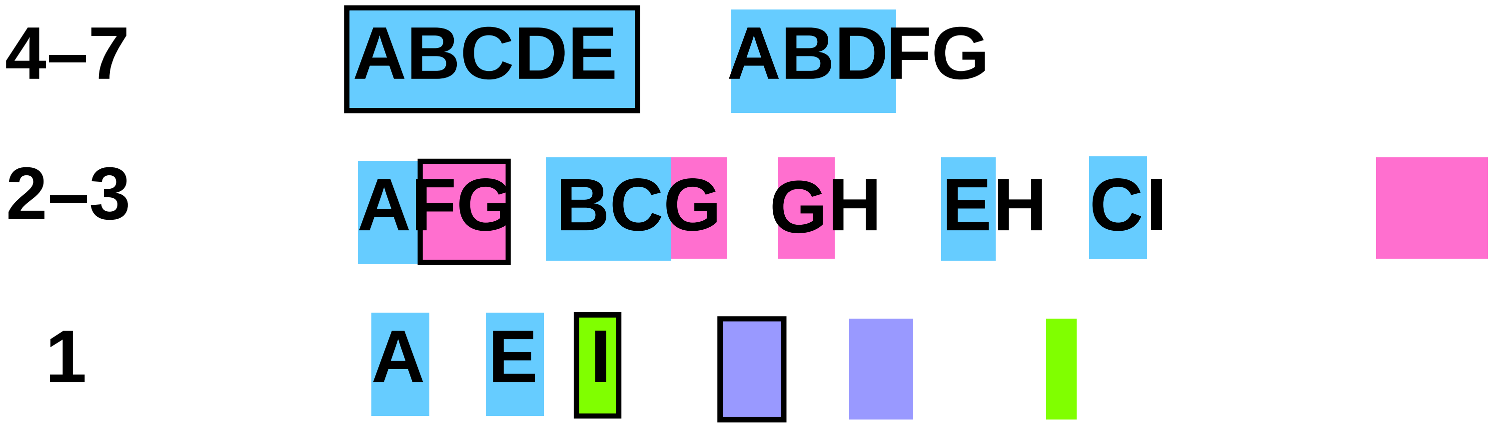
- Huge effort to find max $|S_i - C|$
- Instead find set close to maximum uncovered size
- If always at least factor α × maximum:
  - We have $1 + (\ln n) / \alpha$ approximation algorithm
  - Proof similar to that for greedy

- We call it Disk-Friendly Greedy (DFG)

How to achieve this

- Select parameter $p > 1$: governs approximation and run time
- Partition sets into subcollections:
  - S_i in Z_k if: $p^k \le |S_i| < p^{k+1}$
- For $k \leftarrow K$ down to 0:
  - For each set S_i in Z_k:
    - If $|S_i - C| \ge p^k$: select S_i and update C
    - Else: let $S_i \leftarrow S_i - C$ and add it to $Z_{k'}$: $p^{k'} \le |S_i| < p^{k'+1}$
- For each S_i in Z_0: select S_i, update C, if has uncovered item

Example DFG run



In-memory Cost analysis

- Each S_i either selected or put in lower subcollection
- Guaranteed to shrink by factor p every other pass
- Total number of items in all iterations is $(1 + 1/(p-1))|S_i|$
- So $1 + 1/(p-1)$ times input read time

Disk model analysis

- All file accesses are sequential!
- Initial sweep through input
- Two passes for each subcollection
  - One when sets from higher subcollections added
  - One to select or knock down sets
- Block size B, K collections:
  - Disk accesses for reading input: $D = \sum|S_i| / B$
  - DFG requires $2D[1 + 1/(p-1)] + 2K$ disk reads

Disk-based results

- Tested on Frequent Itemset Mining Dataset Repository
- Show results on kosarak (31Mb) and webdocs (1.4Gb)

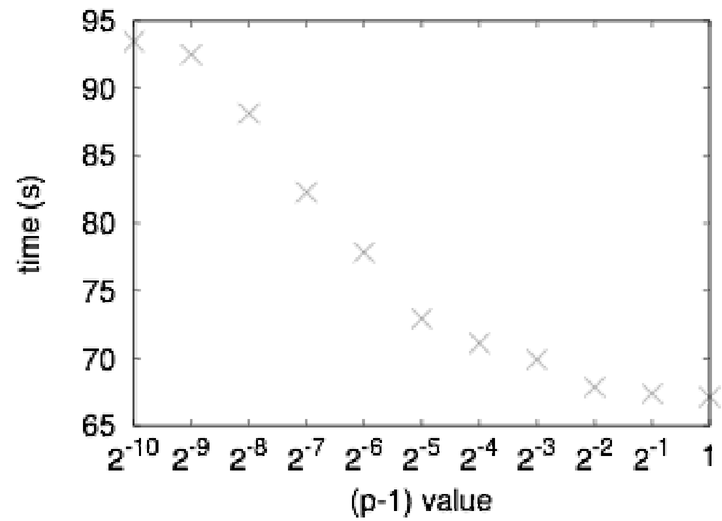
		time (s)	|Solution|
kosarak.dat	naive	8.51	20664
	multipass	331.66	17746
	greedy	98.66	17750
	DFG	2.61	17748
webdocs.dat	naive	91.21	433412
	multipass	—	—
	greedy	—	—
	DFG	86.28	406440

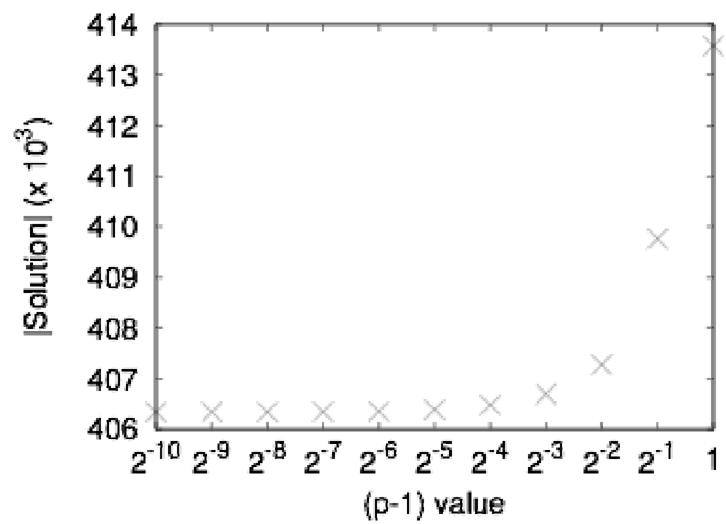
Memory-based results

		time (s)	|Solution|
kosarak.dat	naive	2.20	20664
	multipass	4.21	17746
	greedy	2.99	17750
	DFG	1.97	17741
webdocs.dat	naive	100.98	433412
	multipass	8049.08	406381
	greedy	199.02	406351
	DFG	93.38	406338

Impact of *p*

- RAM-based results for webdocs.dat
- Improving guaranteed accuracy only increases running time by 50% (30s)
- Observed solution size improves, though not as much





Summary

- Noted poor performance of greedy, especially on disk
- Introduced alternative algorithm to greedy:
  - Has approximation bound similar to greedy
- On each disk-resident dataset: our algorithm 10 × faster
- On largest instance: over 400 × faster
- Solution essentially as good as greedy
- Disk version almost as fast as RAM version:
  - Not disk bound!